

# General issues with the implementation of theory models in generators

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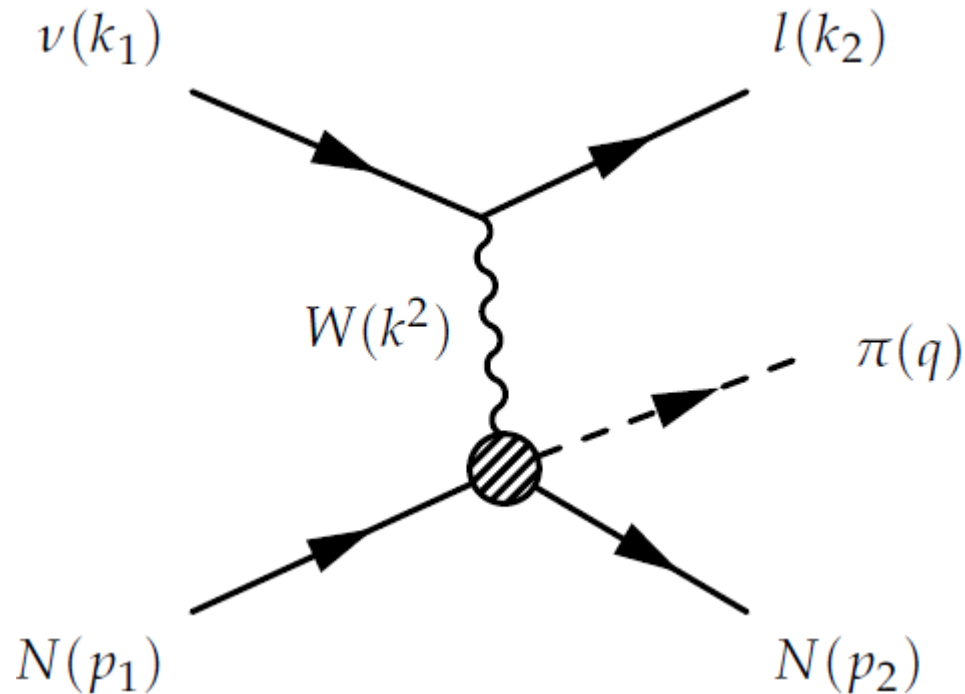
# Outline

- I. Nucleon complexity
- II. Nuclear complexity
- III. Final state interaction

Underlying message:

More exclusive signals → higher dimensional problems

# $\nu + N \rightarrow \pi + N + l$ : counting variables



5 Four vectors =  $5 \times 4 = 20$  variables

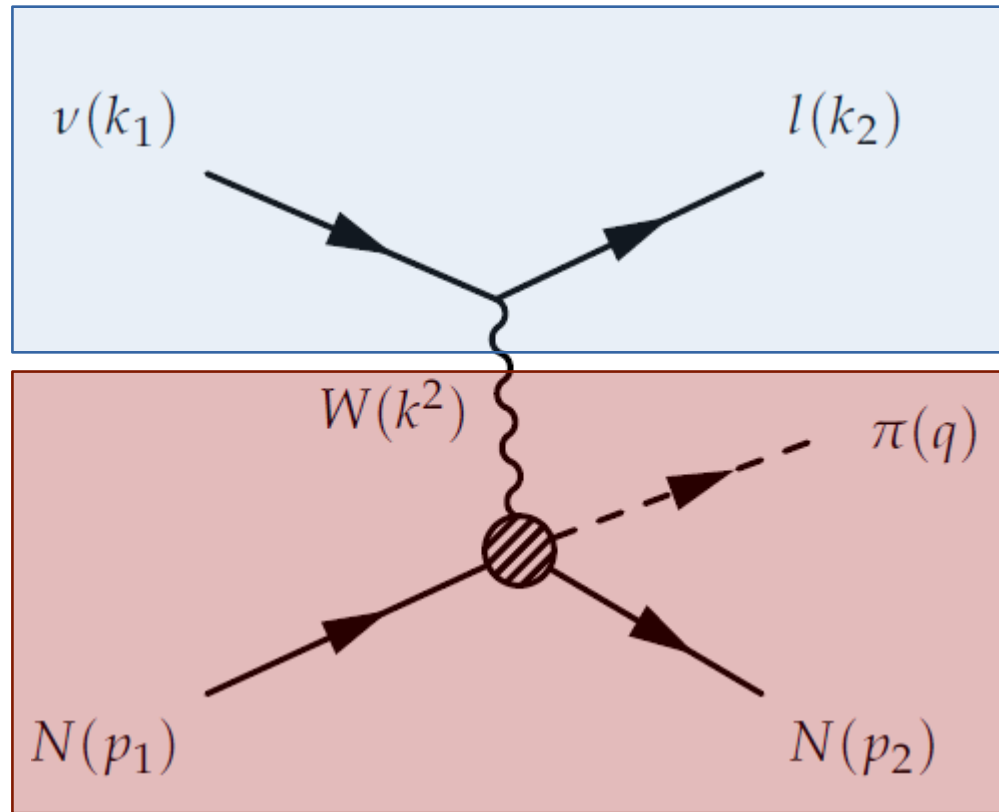
- 4 : on mass shell relations
- 4 : initial nucleon known (at rest)
- 4 : Energy-momentum conservation
- 3 : Freedom to choose reference frame

And invariance along  $q$   
(known direction of one four vector)

= 5 independent variables

$$E_\nu, \cos\theta_l, E_l, \Omega_\pi^* \quad \text{or} \quad E_\nu, Q^2, W, \Omega_\pi^*$$

# $\nu + N \rightarrow \pi + N + l$ : Born approximation



$$\sigma \propto L^{\mu\nu}(k_1, k_2) \times H_{\mu\nu}(k, q, p_2)$$

Leptonic part (PW approximation) → known

Hadronic part → modelling effort

Exploit these facts:

- Lepton tensor is known
- Hadronic part is invariant under rotation along  $q$  and is the product of Hadronic current with its conjugate

→ Separate the  $\phi^*$  dependence

$$\frac{d\sigma}{dQ^2 dW d\Omega_{\pi}^*} = \frac{\mathcal{F}^2}{(2\pi)^4} \frac{k_{\pi}^*}{k_l^2} \times [A + B \cos(\phi^*) C \cos(2\phi^*) + D \sin(\phi^*) + E \sin(2\phi^*)]$$

# Separating the variables

Example for the A structure function:

$$A = L^{00}H_{00} + 2L^{30}H_{30}^s + L^{33}H_{33} + \frac{L^{11} + L^{22}}{2} (H_{11} + H_{22}) + 2iL^{12}H_{12}^a$$

Here the Hadron tensor depends on 3 variables:

$W$ ,  $Q^2$ ,  $\cos\theta_\pi^*$  and  $\varphi_\pi^* = 0$

And in total one needs 15 elements of the hadron tensor

For inclusive:

Only A survives integration over pion angles:

$$\frac{d\sigma}{dQ^2 dW} = \frac{\mathcal{F}}{(2\pi)^4} \frac{k_\pi^*}{k_l^2} \times \left[ L^{00}W_{CC} + 2L^{30}W_{CL} + L^{33}W_{LL} + \frac{L^{11} + L^{22}}{2} (W_T) + iL^{12}W_{T'} \right]$$

And responses depend on  $Q^2$  and  $W$

$$\frac{d\sigma}{dQ^2 dW d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4} \frac{k_\pi^*}{k_l^2} \times [A + B \cos(\phi^*) C \cos(2\phi^*) + D \sin(\phi^*) + E \sin(2\phi^*)]$$

# What we know from electro- and photoproduction

Many approaches in the literature:

- MAID07 -DCC ( e.g. Sato and Lee) -Effective Lagrangian approaches, ChPT , ...

## Ingredients:

- Nucleon resonances
- Background terms : Born term, Vector meson exchanges
- cross channel resonances
- Final state interactions
- ...
- Many parameters fitted to > 20000 datapoints:

# What we know

Many approaches  
-MAID07 -DC

## Ingredients

**Table 12.** The proton parameters  $\alpha$  and  $\beta$  as defined by eq. (47), in units of the longitudinal amplitude at  $Q^2 = 0$  and the values for the transverse amplitudes  $A_{1/2}$  and  $A_{3/2}$  by the real photon physics and

Proton	$A_{1/2}$		
	$\alpha$	$\beta$	
$D_{13}(1520)$	7.77	1.09	0
$S_{11}(1535)$	1.61	0.70	
$S_{31}(1620)$	1.86	2.50	
$S_{11}(1650)$	1.45	0.62	
$D_{15}(1675)$	0.10	2.00	0
$F_{15}(1680)$	3.98	1.20	1
$D_{33}(1700)$	1.91	1.77	1
$P_{13}(1720)$	1.89	1.55	1

**Table 13.** The neutron parameters  $\alpha$  and  $\beta$  as defined by eq. (47), in units of the longitudinal amplitude at  $Q^2 = 0$  and the values for the transverse amplitudes  $A_{1/2}$  and  $A_{3/2}$  by the real photon physics and

Neutron	$A_{1/2}$		$A_{3/2}$		$S_{1/2}$		$S_{1/2}^0$
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	
$D_{13}(1520)$	-0.53	1.55	0.58	1.75	15.7	1.57	13.6
$S_{11}(1535)$	4.75	1.69	—	—	0.36	1.55	28.5
$S_{11}(1650)$	0.13	1.55	—	—	-0.50	1.55	10.1
$D_{15}(1675)$	0.01	2.00	0.01	2.00	0.00	0.00	0.00
$F_{15}(1680)$	0.00	1.20	4.09	1.75	0.00	0.00	0.00
$P_{13}(1720)$	12.7	1.55	4.99	1.55	0.00	0.00	0.00

**Table 5.** Masses and coupling constants for vector mesons, PS-PV mixing parameter  $\Lambda_m$ , and parameter  $A$  for the low-energy correction of eq. (16).

	$m_V$ [MeV]	$\lambda_V$	$\tilde{g}_{V1}$	$\tilde{g}_{V2}/\tilde{g}_{V1}$
$\omega$	783	0.314	16.3	-0.94
$\rho$	770	0.103	1.8	12.7
$\Lambda_m = 423 \text{ MeV}$		$A = 1.9 \times 10^{-3}/m_\pi^+$		$B = 0.71 \text{ fm}$

**Table 6.** Resonance masses  $M_R$ , widths  $\Gamma_R$ , single-pion branching ratios  $\beta_\pi$ , and angles  $\phi_R$  as well as the parameters  $X_R$ ,  $n_E$ , and  $n_M$  of the vertex function eq. (21).

$N^*$	$M_R$ [MeV]	$\Gamma_R$ [MeV]	$\beta_\pi$	$\phi_R$ [deg]	$X_R$ [MeV]	Proton		Neutron	
						$n_E$	$n_M$	$n_E$	$n_M$
$P_{33}(1232)$	1232	130	1.0	0.0	570	-1	2	-1	2
$P_{11}(1440)$	1440	350	0.70	-15	470	—	0	—	-1
$D_{13}(1520)$	1530	130	0.60	32	500	3	4	7	2
$S_{11}(1535)$	1535	100	0.40	8.2	500	2	—	2	—
$S_{31}(1620)$	1620	150	0.25	23	470	5	—	5	—
$S_{11}(1650)$	1690	100	0.85	7.0	500	4	—	4	—
$D_{15}(1675)$	1675	150	0.45	20	500	3	5	3	4
$F_{15}(1680)$	1680	135	0.70	10	500	3	3	2	2
$D_{33}(1700)$	1740	450	0.15	61	700	4	5	4	5
$P_{13}(1720)$	1740	250	0.20	0.0	500	3	3	3	3
$F_{35}(1905)$	1905	350	0.10	40	500	4	5	4	5
$P_{31}(1910)$	1910	250	0.25	35	500	—	1	—	1
$F_{37}(1950)$	1945	280	0.40	30	500	6	6	6	6

		PDG	GW06	2003	2007
$P_{33}(1232)$	$A_{1/2}$	$-135 \pm 6$	$-139.1 \pm 3.6$	-140	-140
	$A_{3/2}$	$-250 \pm 8$	$-257.6 \pm 4.6$	-265	-265
$E2/M1$ (%)		$-2.5 \pm 0.5$		-2.2	-2.2
$P_{11}(1440)$	$A_{1/2}$	$-65 \pm 4$	$-50.6 \pm 1.9$	-77	-61
$D_{13}(1520)$	$A_{1/2}$	$-24 \pm 9$	$-28.0 \pm 1.9$	-30	-27
	$A_{3/2}$	$166 \pm 5$	$143.1 \pm 2.0$	166	161
$S_{11}(1535)$	$A_{1/2}$	$90 \pm 30$	$91.0 \pm 2.2$	73	66
$S_{31}(1620)$	$A_{1/2}$	$27 \pm 11$	$49.6 \pm 2.2$	71	66
$S_{11}(1650)$	$A_{1/2}$	$53 \pm 16$	$22.2 \pm 7.2$	32	33
$D_{15}(1675)$	$A_{1/2}$	$19 \pm 8$	$18.0 \pm 2.3$	23	15
	$A_{3/2}$	$15 \pm 9$	$21.2 \pm 1.4$	24	22
$F_{15}(1680)$	$A_{1/2}$	$-15 \pm 6$	$-17.3 \pm 1.4$	-25	-25
	$A_{3/2}$	$133 \pm 12$	$133.6 \pm 1.6$	134	134
$D_{33}(1700)$	$A_{1/2}$	$104 \pm 15$	$125.4 \pm 3.0$	135	226
	$A_{3/2}$	$85 \pm 22$	$105.0 \pm 3.2$	213	210
$P_{13}(1720)$	$A_{1/2}$	$18 \pm 30$	$96.6 \pm 3.4$	55	73
	$A_{3/2}$	$-19 \pm 20$	$-39.0 \pm 3.2$	-32	-11
$F_{35}(1905)$	$A_{1/2}$	$26 \pm 11$	$21.3 \pm 3.6$	14	18
	$A_{3/2}$	$-45 \pm 20$	$-45.6 \pm 4.7$	-22	-28
$F_{37}(1950)$	$A_{1/2}$	$-76 \pm 12$		-78	-94
	$A_{3/2}$	$-97 \pm 10$		-101	-121

**Table 8.** Neutron helicity amplitudes at  $Q^2 = 0$  for the major nucleon resonances. GW02 are the results GWU/SAID analysis [28]. Further notation as in table 7.

		PDG	GW02	2003	2007
$P_{11}(1440)$	$A_{1/2}$	$40 \pm 10$	$47 \pm 5$	52	54
$D_{13}(1520)$	$A_{1/2}$	$-59 \pm 9$	$-67 \pm 4$	-85	-77
	$A_{3/2}$	$-139 \pm 11$	$-112 \pm 3$	-148	-154
$S_{11}(1535)$	$A_{1/2}$	$-46 \pm 27$	$-16 \pm 5$	-42	-51
$S_{11}(1650)$	$A_{1/2}$	$-15 \pm 21$	$-28 \pm 4$	27	9
$D_{15}(1675)$	$A_{1/2}$	$-43 \pm 12$	$-50 \pm 4$	-61	-62
	$A_{3/2}$	$-58 \pm 13$	$-71 \pm 5$	-74	-84
$F_{15}(1680)$	$A_{1/2}$	$29 \pm 10$	$29 \pm 6$	25	28
	$A_{3/2}$	$-33 \pm 9$	$-58 \pm 9$	-35	-38
$P_{13}(1720)$	$A_{1/2}$	$1 \pm 15$		17	-3
	$A_{3/2}$	$-29 \pm 61$		-75	-31

Table 5. Masses and coupling constants for vector mesons, PS-PV mixing parameter  $\Lambda_m$ , and parameter  $A$  for the low-energy correction of eq. (16).

- MAID07
- DCC ( e.g. Sato and Lee)
- Effective Lagrangian approaches, ...

Table 6. Resonance masses  $M_R$ , widths  $\Gamma_R$ , single-pion

## -Nucleon resonances

-Background terms : Born term, Vector meson exchanges

- Final state interactions

Many parameters fitted to > 20000 datapoints:

For neutrinos no such dataset is available

**Table 13.** The neutron parameters  $E_{\text{eff}}^{\text{neut}}(1950)$  and  $1945$  on  $^{289}\text{Fr}$ . The values for the transverse amplitudes  $A_{1/2,3/2}^0$  are given in table 8. Further notation as in table 12.

	$A_{1/2}$		$A_{3/2}$		$S_{1/2}$		$S_{1/2}^0$
Neutron	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	
$D_{13}(1520)$	-0.53	1.55	0.58	1.75	15.7	1.57	13.6
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# Electroproduction data

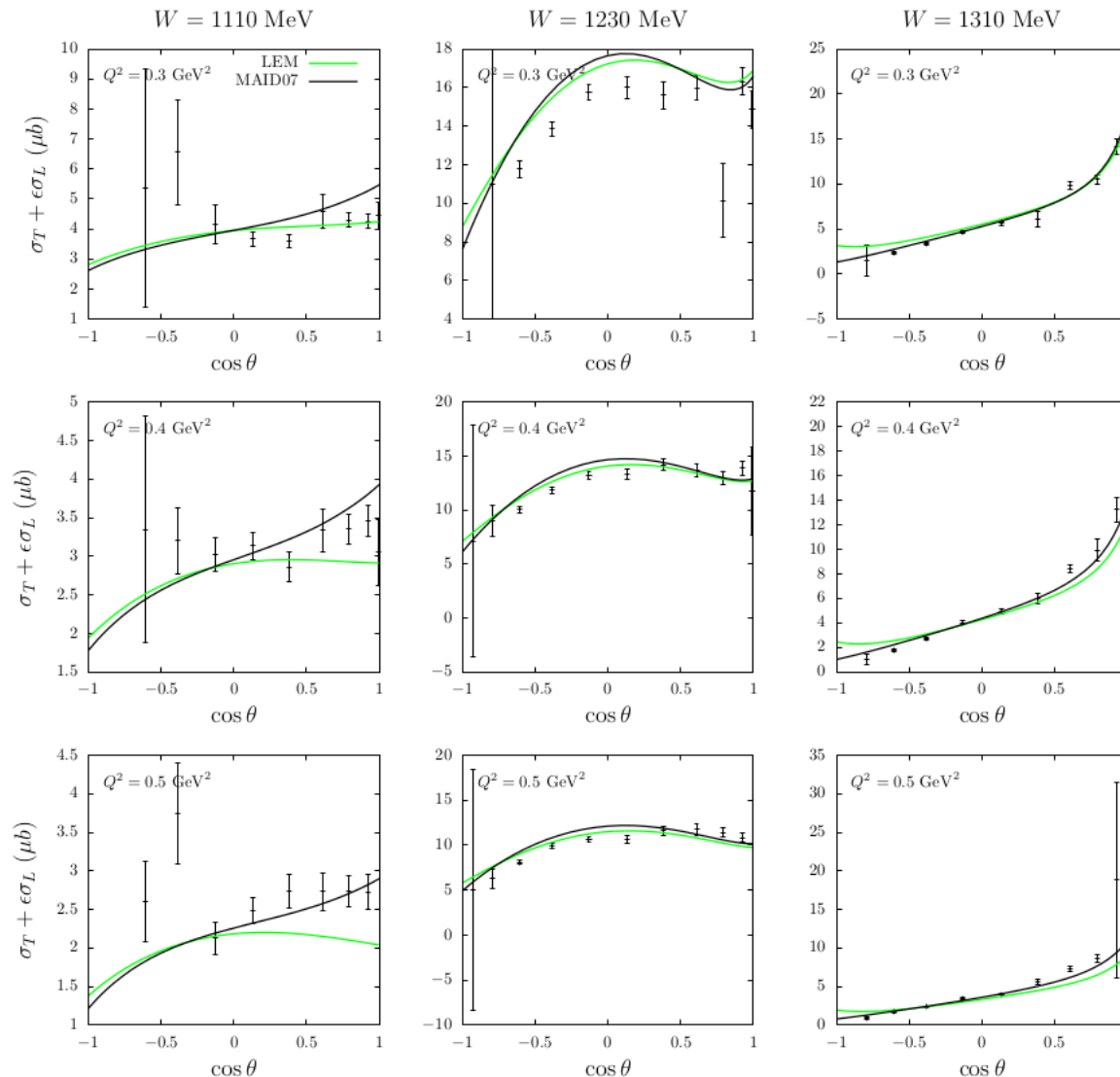
$$\frac{d\sigma_\nu}{dW dQ^2 d\Omega^*} = \frac{\mathcal{F}^2}{(2\pi)^4} \frac{k_\pi^*}{k_l^2} \times [A + B \cos(\phi^*) + C \cos(2\phi^*) + D \sin(\phi^*) + E \sin(2\phi^*)]$$

Write lepton tensor for polarized electron explicitly

$$\frac{d\sigma_e}{d\Omega^*} = \sigma_T + \epsilon \sigma_L + \sqrt{2\epsilon(1+\epsilon)} \sigma_{LT} \cos(\phi^*) + \epsilon \sigma_{TT} \cos(2\phi^*) + h \sqrt{2\epsilon(1-\epsilon)} \sigma_{LT'} \sin \phi^*$$

# Electroproduction data: $e+p \rightarrow n + \pi^+$

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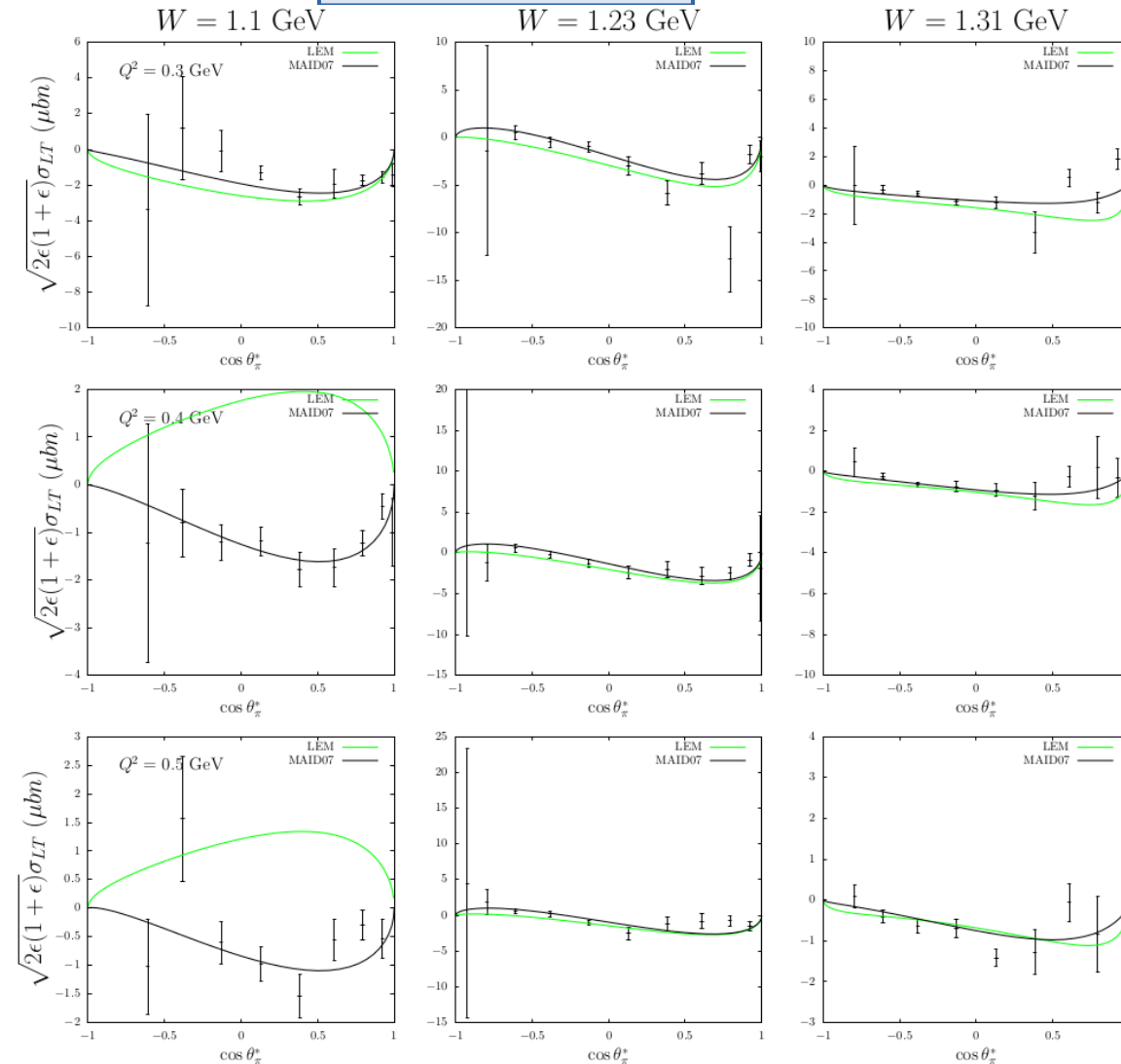
LEM from R. Gonzalez-Jimenez et al.  
Phys. Rev. D 95, 113007 (2017)  
Based on HNV model

Data from E89-038 CLAS  
experiment, 1999, V. Burkert, R.  
Minehart

MAID07 :  
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34: 69

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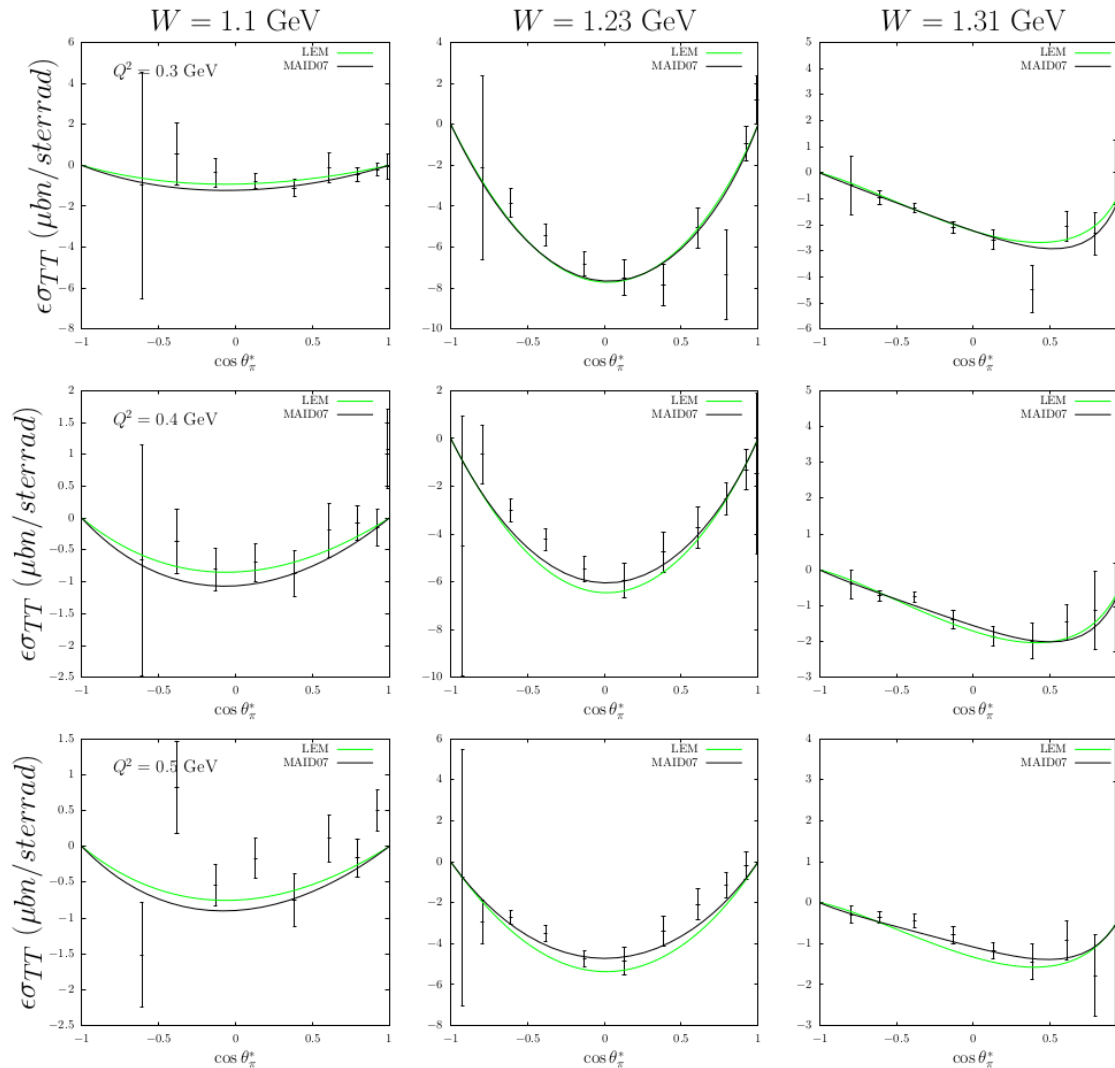
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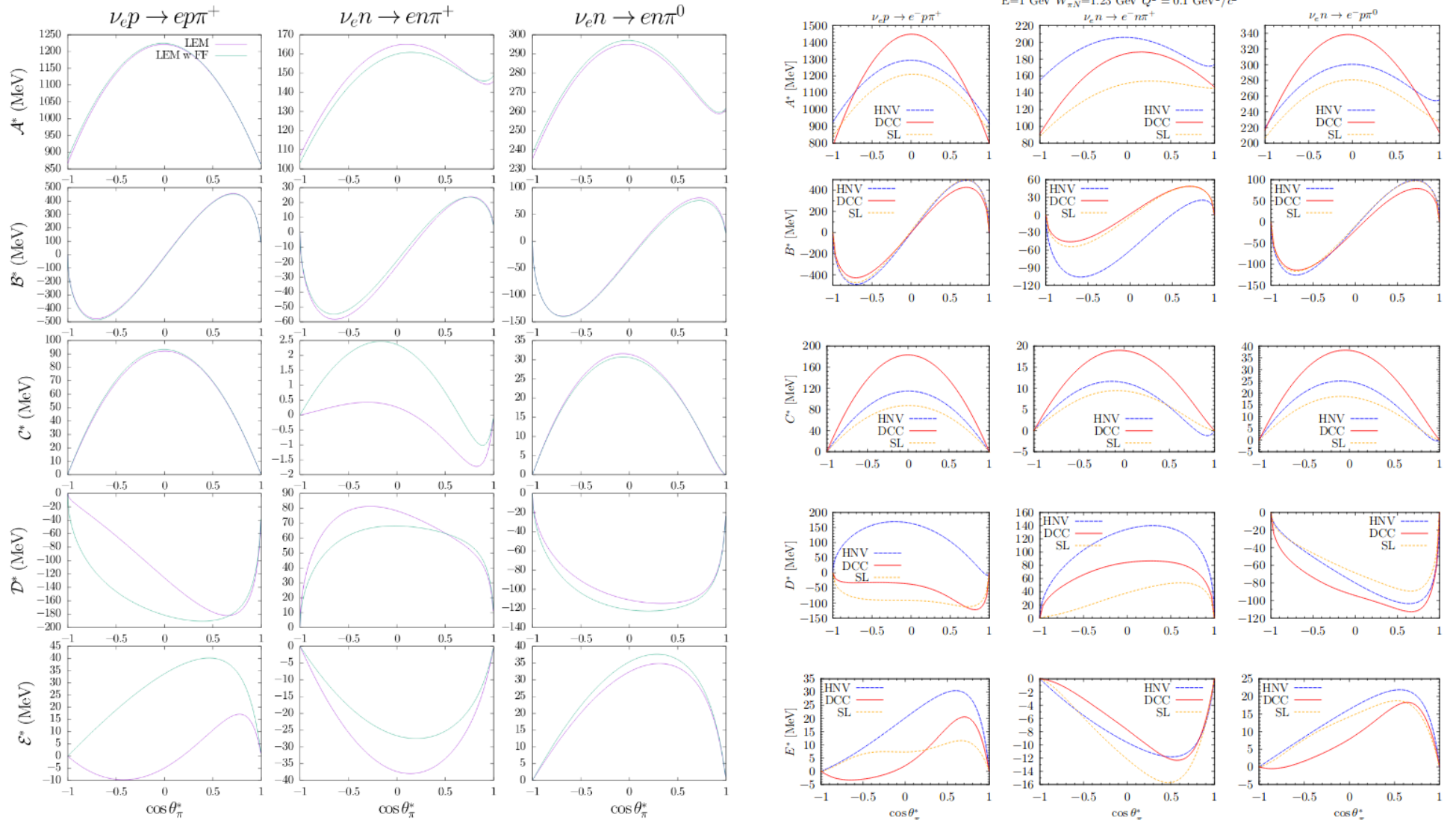


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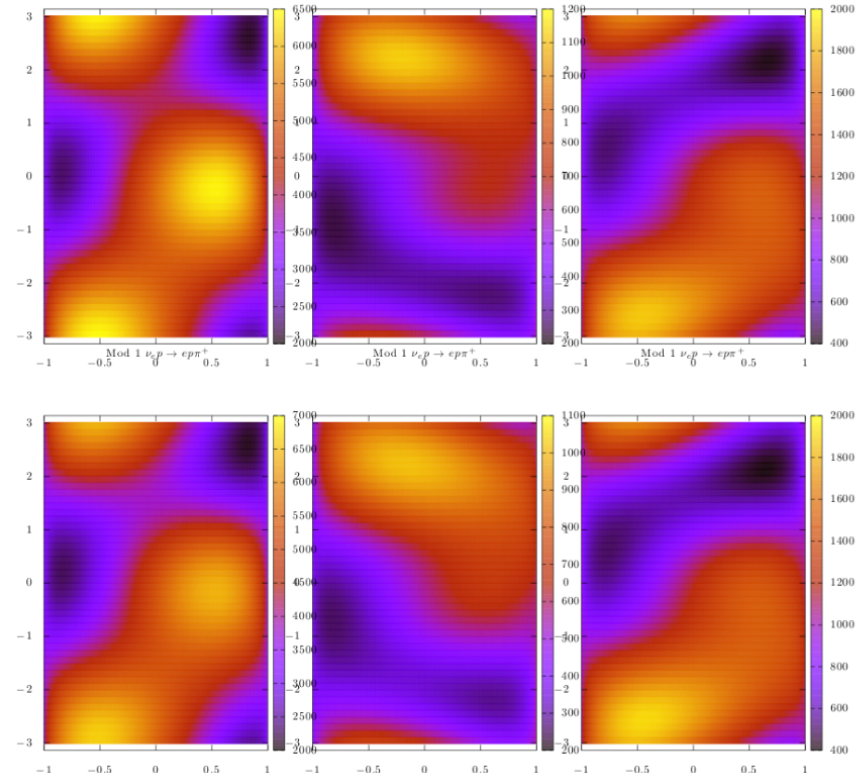
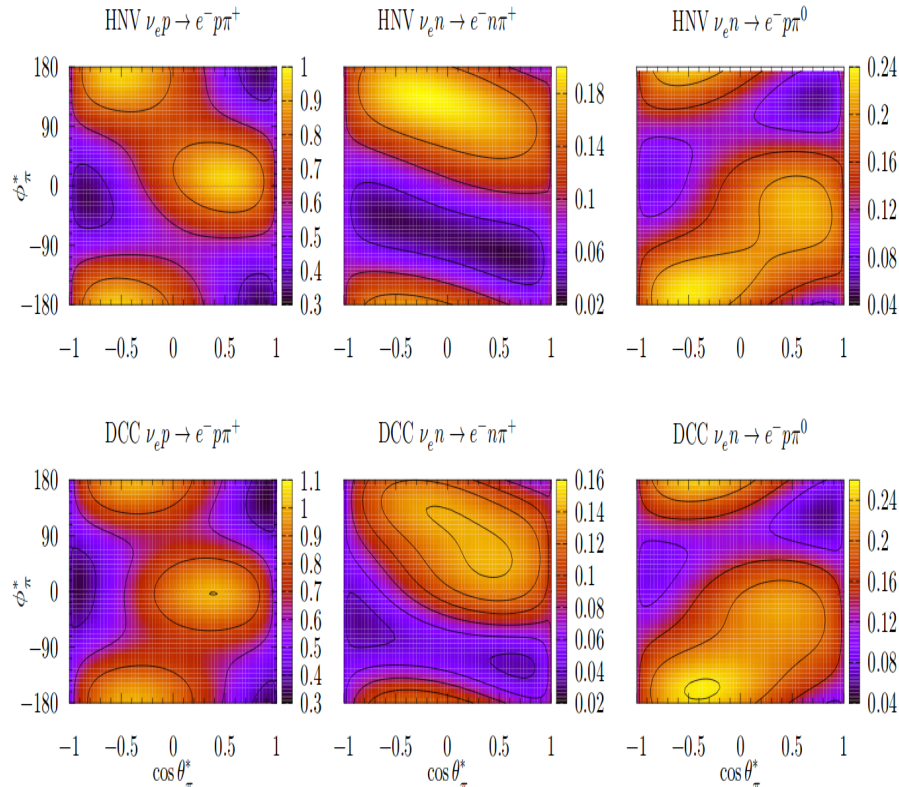
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# Structure functions for neutrinos



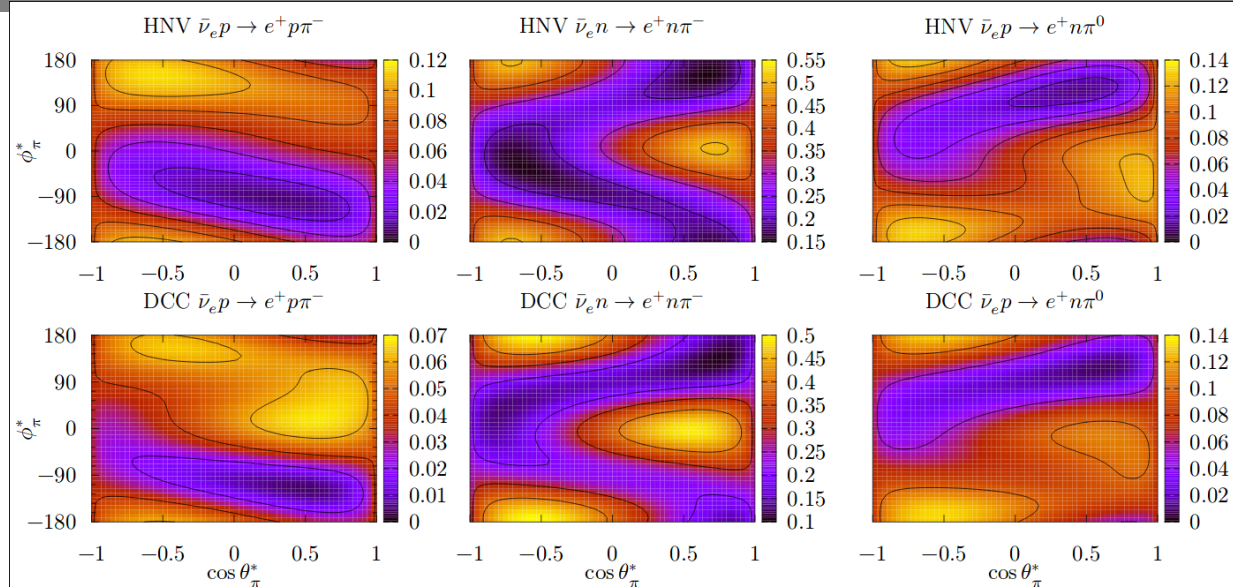
# Angular distributions for neutrinos



HNV, DCC and LEM vary in structure functions, still more or less agree on angular cross section. (Around Delta peak)

Could this influence neutrino oscillation analysis ?

# Angular distributions for neutrinos



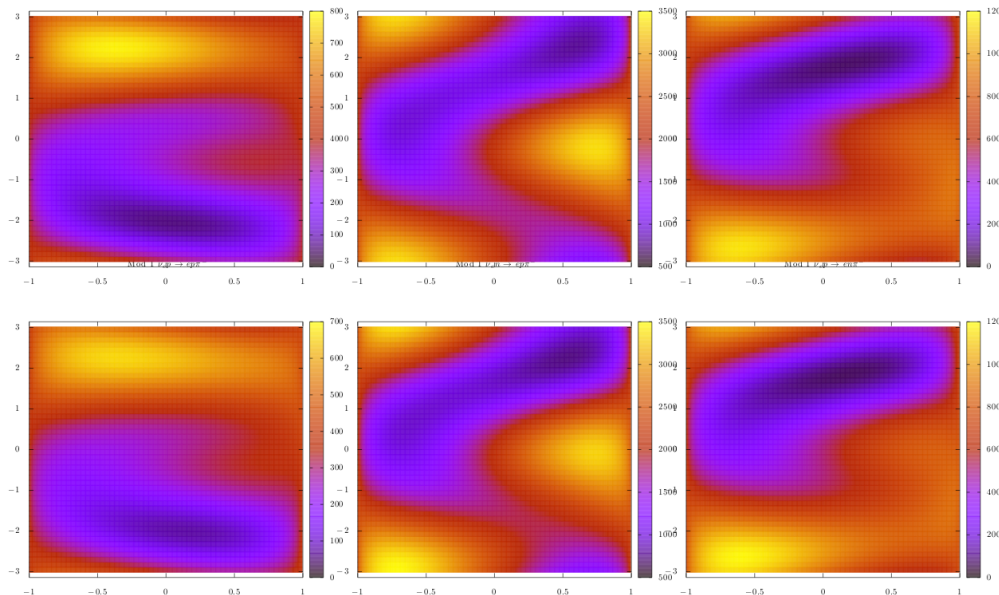
In (most) event generators:

Isotropic distribution in CMS.  
→ Computationally easy

What is the difficulty ?

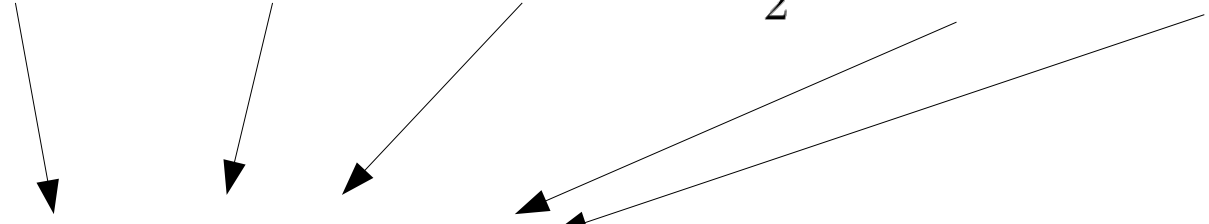
× Time to compute cross section  
→ Actually rather fast

The problem is efficiency in  
Sampling the phase space



# How to introduce the fivefold CS ?

Sample inclusive cross section in the traditional way:

$$\frac{d\sigma}{dQ^2 dW} = \frac{\mathcal{F}}{(2\pi)^4} \frac{k_\pi^*}{k_l^2} \times \left[ L^{00} W_{CC} + 2L^{30} W_{CL} + L^{33} W_{LL} + \frac{L^{11} + L^{22}}{2} (W_T) + iL^{12} W_{T'} \right]$$
The diagram shows five arrows originating from the terms inside the brackets of the equation:  $L^{00}W_{CC}$ ,  $2L^{30}W_{CL}$ ,  $L^{33}W_{LL}$ ,  $\frac{L^{11} + L^{22}}{2}(W_T)$ , and  $iL^{12}W_{T'}$ . These arrows point downwards towards the text 'Tabulate or Calculate in situ inclusive structure functions for the interaction'.

Tabulate or Calculate in situ inclusive structure functions for the interaction



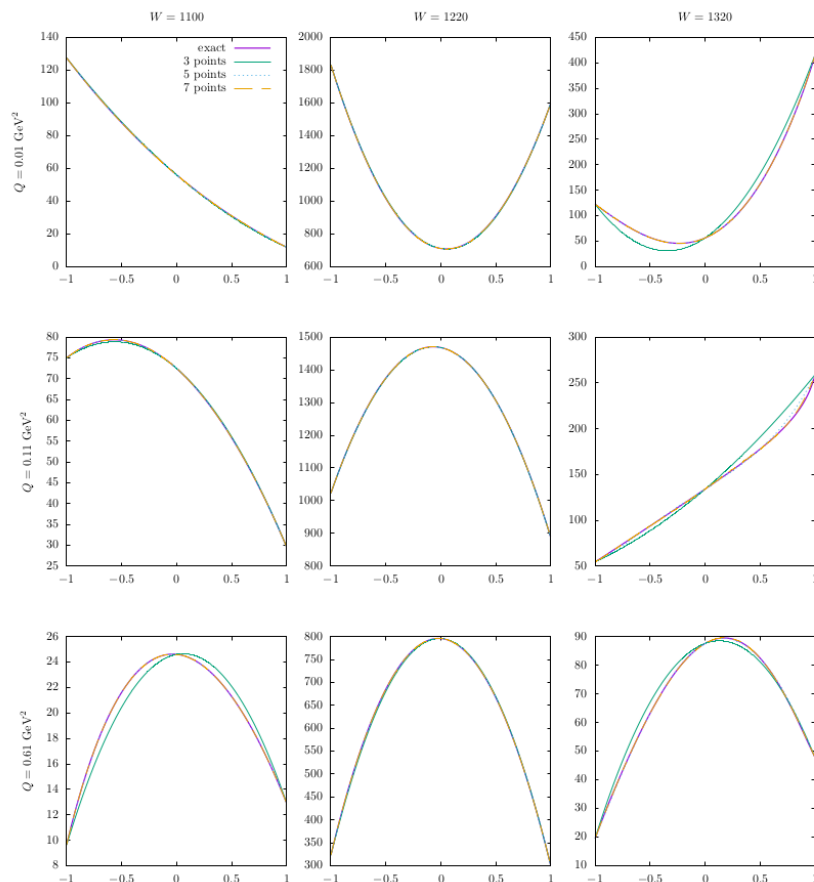
Functions only of  $Q^2$  and  $W$ , very fast interpolation in 2D.

This gives an event with  $Q^2$  and  $W$

# How to introduce the fivefold CS ?

given a  $Q^2$  and  $W$ , distribution of  $\cos\theta^*$  is determined by A

$$\frac{d\sigma}{dQ^2 dW d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4} \frac{k_\pi^*}{k_l^2} \times [A + B \cos(\phi^*) C \cos(2\phi^*) + D \sin(\phi^*) + E \sin(2\phi^*)]$$

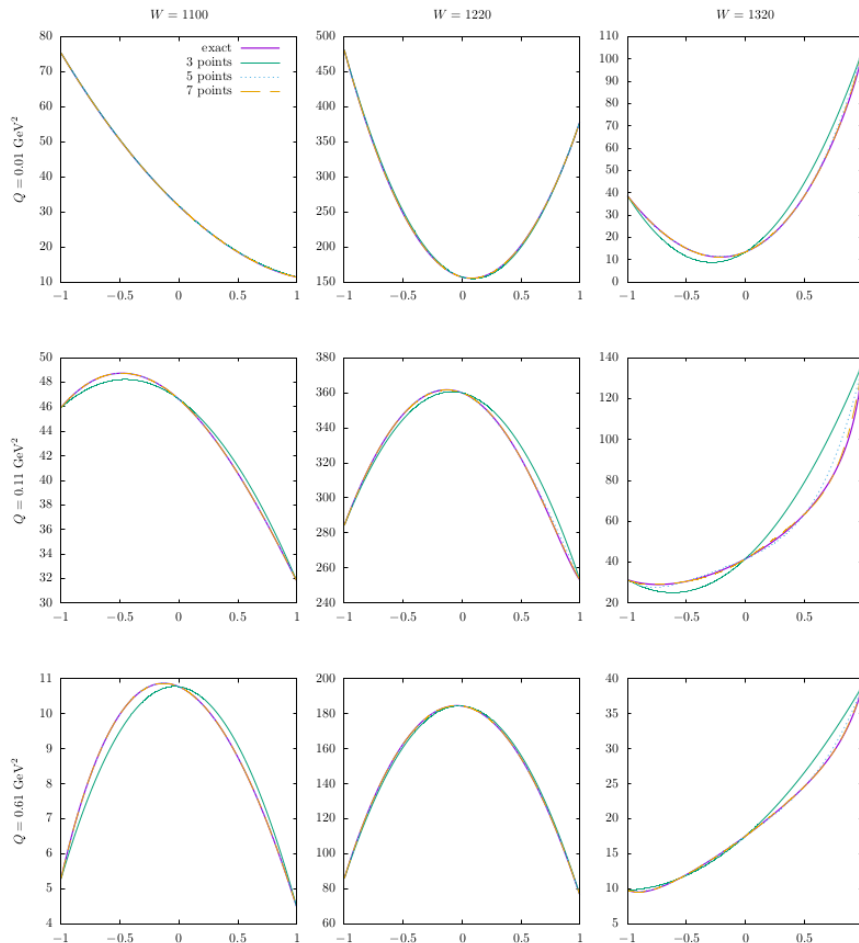


A is a smooth function and can usually be interpolated by a polynomial of degree 2

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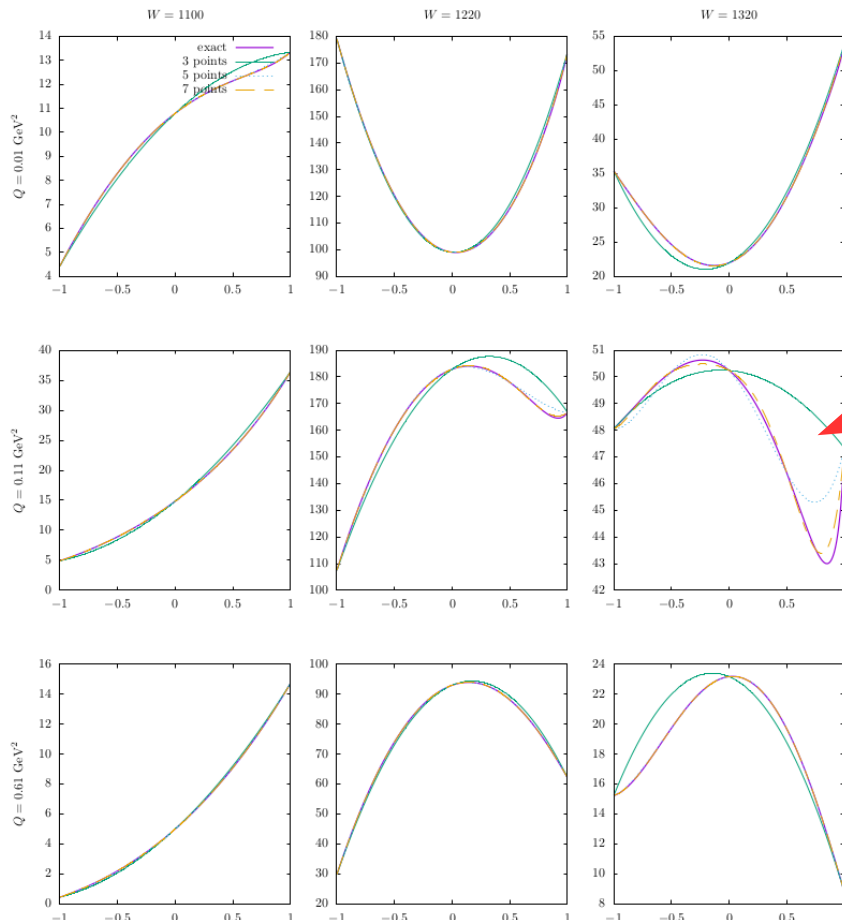


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A is a smooth function and can **usually** be interpolated by a polynomial of degree 2

Calculation of  $A(\cos)$  for fixed  $Q^2$  and  $W$  is very cheap

Interpolation with degree 2 polynomial means:

Cumulative distribution function

$$CDF(\cos(\theta)) = \int a_2 \cos^2 \theta + a_1 \cos \theta + a_0 d \cos \theta$$

Is a monotonic degree 3 polynomial

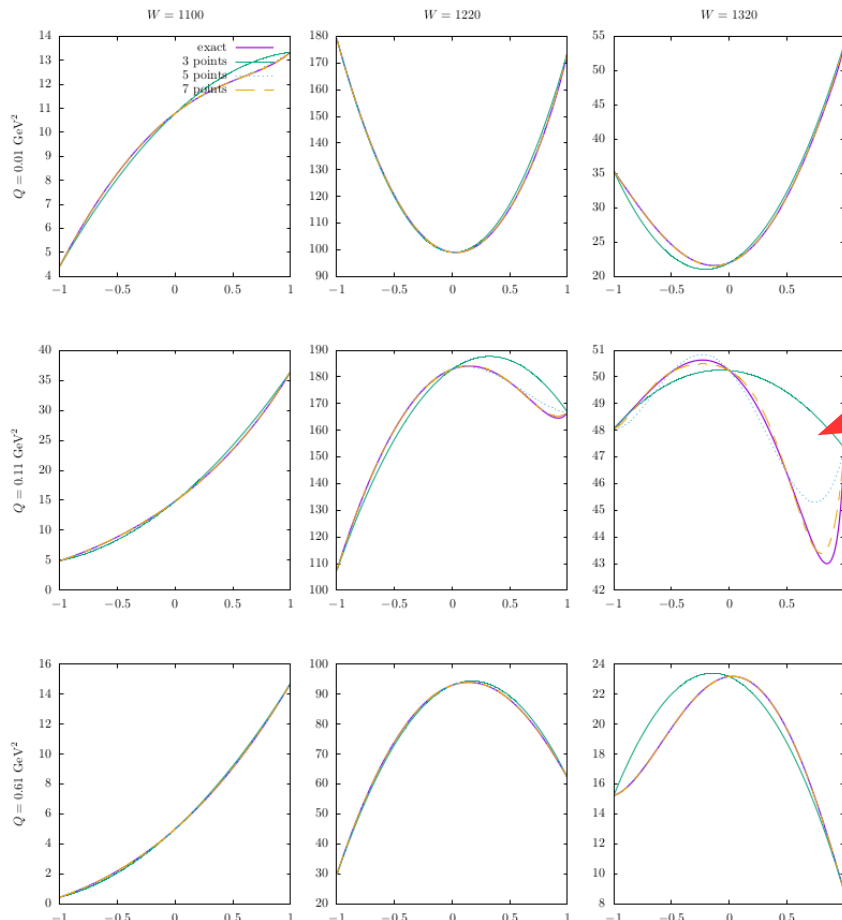
→ Can be inverted analytically

→ Inversion sampling

# How to introduce the fivefold CS ?

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# How to introduce the fivefold CS ?

By calculation of A at 3 points one gets a cosine according to the theoretical distribution  
With efficiency 100%

given a  $Q^2$ ,  $W$ , and  $\cos \theta^*$  distribution of  $\varphi^*$  is

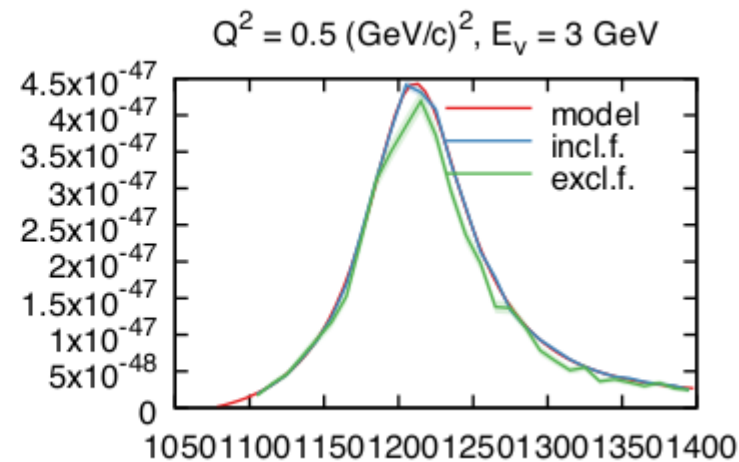
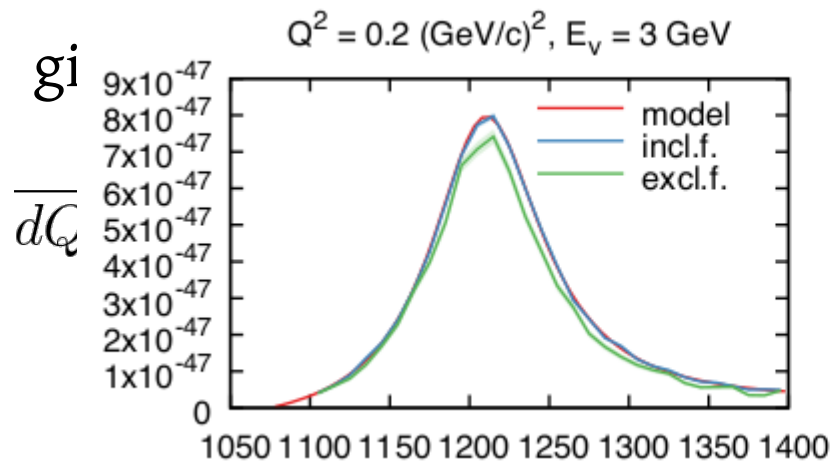
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Again we determine the CDF algebraically.

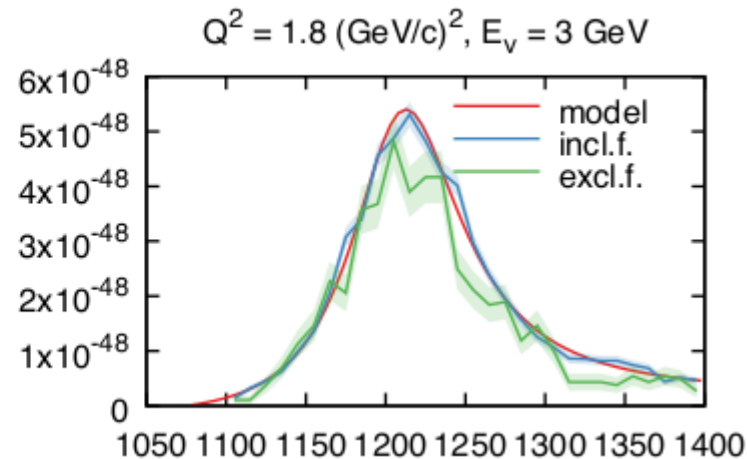
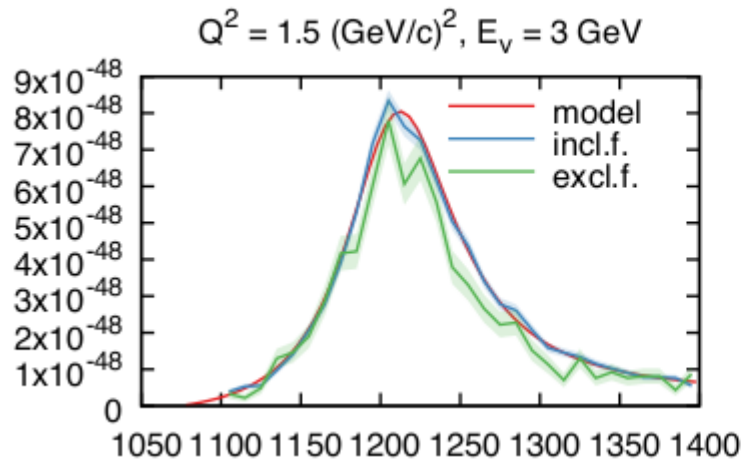
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# How to introduce the fivefold CS ?

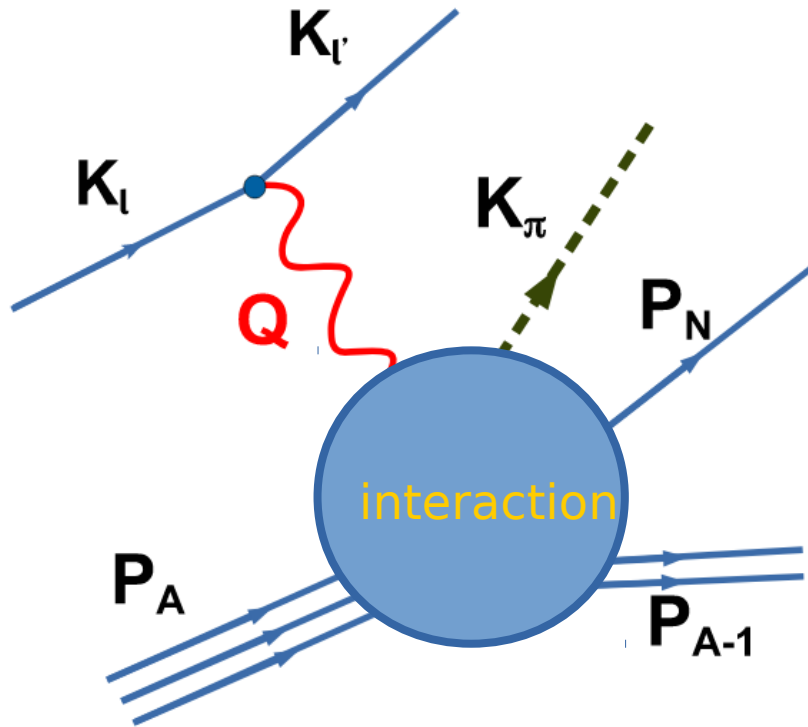
First results, sampling in the full phase space,  
still some issues to be checked and algorithms to be explored



$- E \sin(2\phi^*)]$



# $\nu + A \rightarrow \pi + N + X + l$ : counting variables



6 Four vectors =  $6 \times 4 = 24$  variables

- 4 : on mass shell relations
- 4 : initial nucleus known (at rest)
- 4 : Energy-momentum conservation
- 3 : Freedom to choose reference frame  
And invariance along  $q$   
(known direction of one four vector)

= 9 independent variables

- 1 : Final nucleus left in a hole state  
(i.e. integrate over final nucleus energy)

= 8 independent variables

$$E_\nu, \cos\theta_l, E_l, \Omega_\pi, \Omega_N, k_\pi$$

We go from a  $2 \rightarrow 3$  process to a  $2 \rightarrow 4$  process

But there are no additional constraints because residual nucleus can be in any state.

So from  $5 \rightarrow 9$  variables (one can also interpret the extra 4 variables as four-vector of initial bound nucleon)

# How to introduce the fivefold CS ?

By calculation of A at 3 points one gets a cosine according to the theoretical distribution  
With efficiency 100%

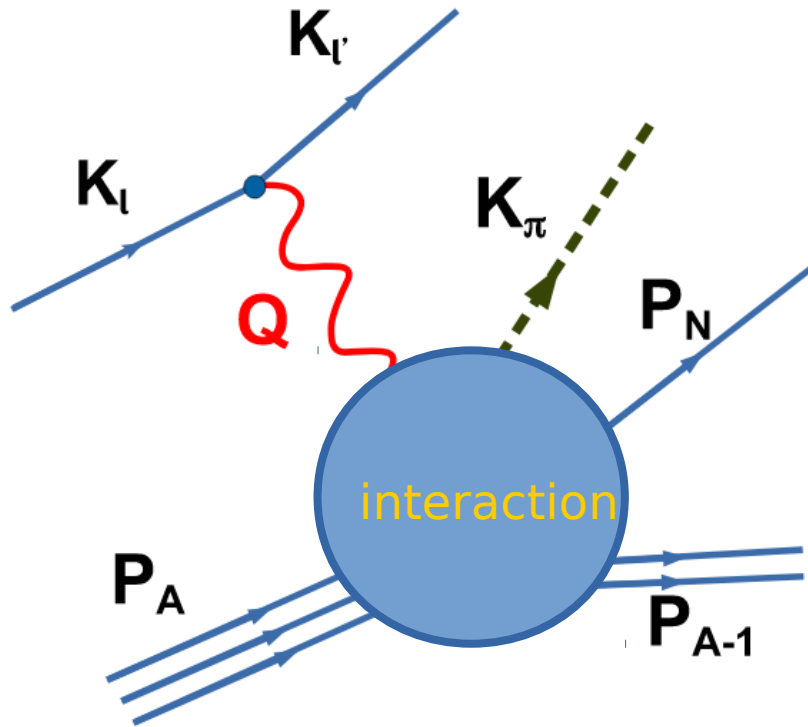
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Again we determine the CDF algebraically.

→ The CDF can be inverted numerically to give  $\varphi^*$

# $\nu + A \rightarrow \pi + N + X + l$ : Born approximation



$$\sigma \propto L^{\mu\nu}(k_1, k_2) \times H_{\mu\nu}(k, q, p_2)$$

$$H_{\mu\nu} = J_\mu^\dagger J_\nu$$

$$J^\mu = \int_X \Psi_f \mathcal{O}^\mu \Psi_i e^{q \cdot r} d\mathbf{r}$$

$\Psi_i$  and  $\Psi_f$  contain the whole initial and final state

Nuclear modeling = finding a good approximation for the wavefunctions

# Impulse approximation

- I. Interaction with only one particle of complex system
- II. The incident particle (Q) is unaffected by the system (in BA)

$$\Psi_{i,f} = \sum \phi_N \otimes \phi_{A-1}$$

Reduces the problem to finding single particle states in nuclear medium:

$$J_{SN}^{\mu} = \int \psi_N \phi_{\pi} \mathcal{O}^{\mu} e^{-i\mathbf{q}\cdot\mathbf{r}} \phi_i d\mathbf{r}$$

# Impulse approximation

- I. Interaction with only one particle of complex system
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Reduces the problem to finding single particle states in nuclear medium:

$$J^\mu = \int d\mathbf{p}'_N \int \frac{d\mathbf{p}}{(2\pi)^{3/2}} \times$$

$$\overline{\psi}_{s_N}(\mathbf{p}'_N, \mathbf{p}_N) \phi^*(\mathbf{k}'_\pi, \mathbf{k}_\pi) \mathcal{O}^\mu_{1\pi}(Q, K'_\pi, P'_N) \psi^{m_j}_\kappa(\mathbf{p}),$$

(6)

With  $\mathbf{p} = \mathbf{p}_m = \mathbf{q} - \mathbf{p}'_N - \mathbf{k}'_\pi$

This is a six dimensional integral with a lot of matrix multiplication...

# Factorization

$$J^\mu = \int d\mathbf{p}'_N \int \frac{d\mathbf{p}}{(2\pi)^{3/2}} \times$$
$$\bar{\psi}_{s_N}(\mathbf{p}'_N, \mathbf{p}_N) \phi^*(\mathbf{k}'_\pi, \mathbf{k}_\pi) \mathcal{O}_{1\pi}^\mu(Q, K'_\pi, P'_N) \psi_\kappa^{m_j}(\mathbf{p}), \quad (6)$$

←  
Replace these by asymptotic momenta

# Relativistic Plane wave Impulse approximation

$$J^\mu = \int d\mathbf{p}'_N \int \frac{d\mathbf{p}}{(2\pi)^{3/2}} \times$$

$$\overline{\psi}_{s_N}(\mathbf{p}'_N, \mathbf{p}_N) \phi^*(\mathbf{k}'_\pi, \mathbf{k}_\pi) \mathcal{O}_{1\pi}^\mu(Q, K'_\pi, P'_N) \psi_\kappa^{m_j}(\mathbf{p}),$$

$$\mathbf{p}'_N = \mathbf{p}_N \quad \mathbf{k}'_\pi = \mathbf{k}_\pi \quad (6)$$

$$H^{\mu\nu} \propto \text{Tr} \left( \psi_b(\mathbf{p}) \overline{\psi}_b(\mathbf{p}) \tilde{\mathcal{O}}^\mu (\not{k}_N + M_N) \mathcal{O}^\nu \right)$$

# Plane wave Impulse approximation

$$H^{\mu\nu} \propto \text{Tr} \left( \psi_b(\mathbf{p}) \bar{\psi}_b(\mathbf{p}) \tilde{\mathcal{O}}^\mu (\not{k}_N + M_N) \mathcal{O}^\nu \right)$$

Projection onto positive energy states

$$H^{\mu\nu} \propto |\psi_b(p)|^2 \text{Tr} \left( (\not{p} + M'_N) \tilde{\mathcal{O}}^\mu (\not{k}_N + M_N) \mathcal{O}^\nu \right)$$

Matrix element becomes proportional to initial momentum distribution

Combination of off-shell plane wave spinor expression  
And probability of finding momentum  $p$  in nucleus

# Plane wave Impulse approximation

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# Plane wave Impulse approximation

Side note:

Difference between RPWIA and PWIA was explored in:

Analysis of factorization in (e,e'p) reactions: A survey of the relativistic plane wave impulse approximation

J.A. Caballero<sup>1,2</sup>, T.W. Donnelly<sup>3</sup>, E. Moya de Guerra<sup>2</sup> and J.M. Udías<sup>4</sup>

Nucl.Phys. A632 (1998) 323-362

No big difference for inclusive responses in CC2 operator

Larger effect for more 'off-shell' operators, and for transverse-longitudinal interference

And probability of finding momentum  $p$  in nucleus

# Plane wave Impulse approximation

$$H^{\mu\nu} \propto \text{Tr} \left( \psi_b(\mathbf{p}) \bar{\psi}_b(\mathbf{p}) \tilde{\mathcal{O}}^\mu (\not{k}_N + M_N) \mathcal{O}^\nu \right)$$

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Matrix element becomes proportional to **initial momentum distribution**

Combination of off-shell plane wave spinor expression  
And probability of finding momentum  $p$  in nucleus

# Plane wave Impulse approximation

$$H^{\mu\nu} \propto |\psi_b(p)|^2 \text{Tr} \left( (\not{p} + M'_N) \tilde{\mathcal{O}}^\mu (\not{k}_N + M_N) \mathcal{O}^\nu \right)$$


Matrix element becomes proportional to  
initial momentum distributions (some examples):

- RFG : plane waves up to  $k_F$
- LFG : plane waves up to  $k_F$  but  $k_F$  depends on nuclear density  $\rightarrow$  possible to introduce additional density dependence
- IPSM : e.g. from mean field (HF/RMF/harmonic oscillator)  $\rightarrow$  different shells have different momentum distribution and separation energies
- IPSM + correlations : account for high momentum components in nuclear momentum distribution

## Nuclear Theory and Event Generators for Charge-Changing Neutrino Reactions

J. W. Van Orden

*Department of Physics, Old Dominion University, Norfolk, VA 23529*

*Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA* 

T. W. Donnelly

*Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics,*

*Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

(Dated: August 5, 2019)

Comparison of these different spectral functions  
For exclusive nucleon knockout  
(RFG, LDA, RMF, Rome model)

# Factorization, with FSI

Transition matrix:

$$\int d\mathbf{p}'_N \int \frac{d\mathbf{p}}{(2\pi)^{3/2}} \psi_{\kappa}^{m_j}(\mathbf{p}) \bar{\psi}_{s_N}(\mathbf{p}'_N, \mathbf{p}_N) \phi^*(\mathbf{k}'_{\pi}, \mathbf{k}_{\pi})$$

In general, dependence on  $q$ ,  $\mathbf{p}_N$  and  $\mathbf{k}_{\pi}$  ( 7 variables )

Contrast with RPWIA : depends only on  $\mathbf{p}_m = \mathbf{p}_N + \mathbf{k}_{\pi} - \mathbf{q}$

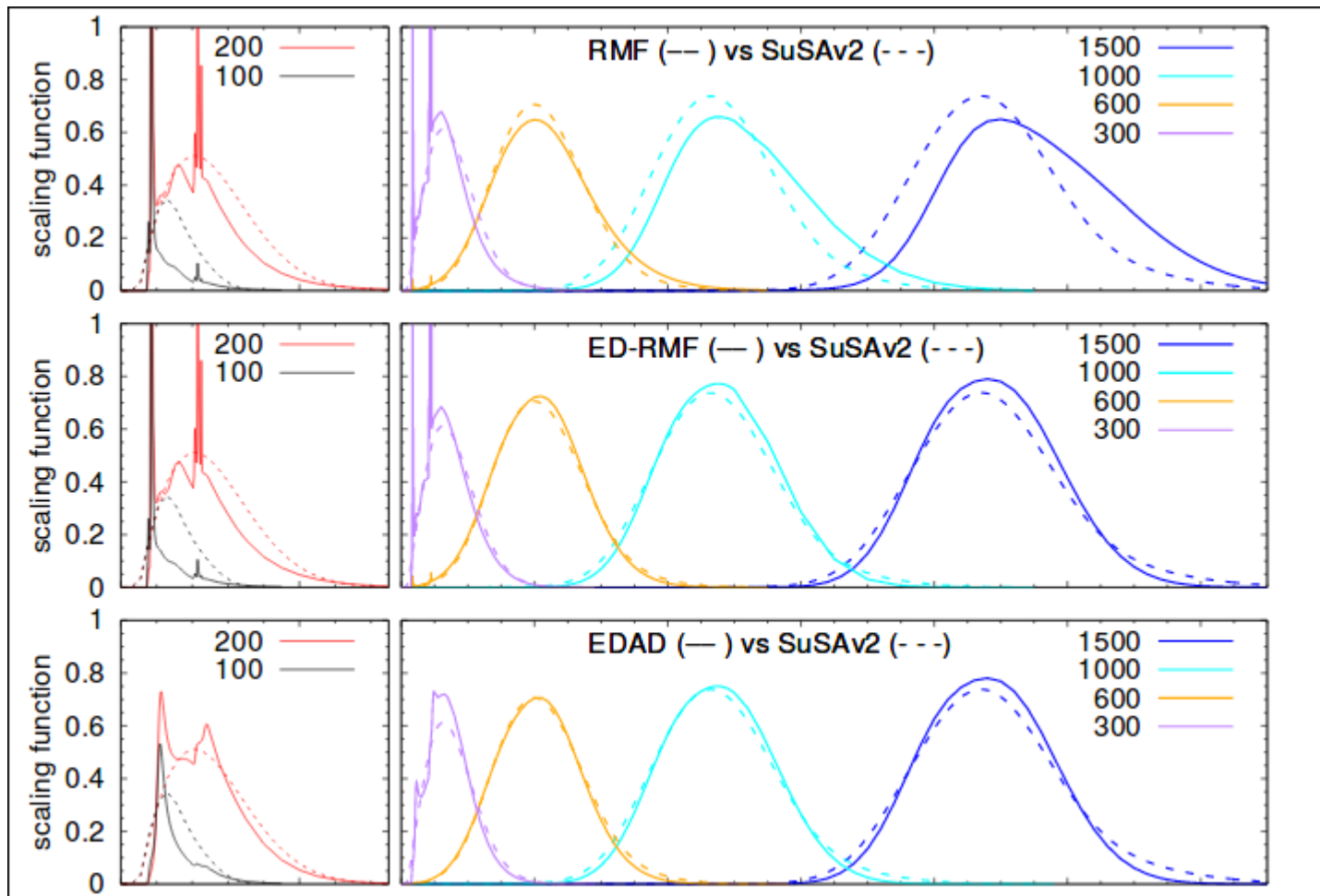
Spreading of the energy momentum relation in a potential

Particles have fixed energy and are only on shell asymptotically  
→ Probing of multiple initial momentum states

# Kinematic dependence

In general, dependence on  $q$ ,  $\mathbf{p}_N$  and  $\mathbf{k}_\pi$  ( 7 variables )

Contrast with RPWIA : depends only on  $\mathbf{p}_m = \mathbf{p}_N + \mathbf{k}_\pi - \mathbf{q}$

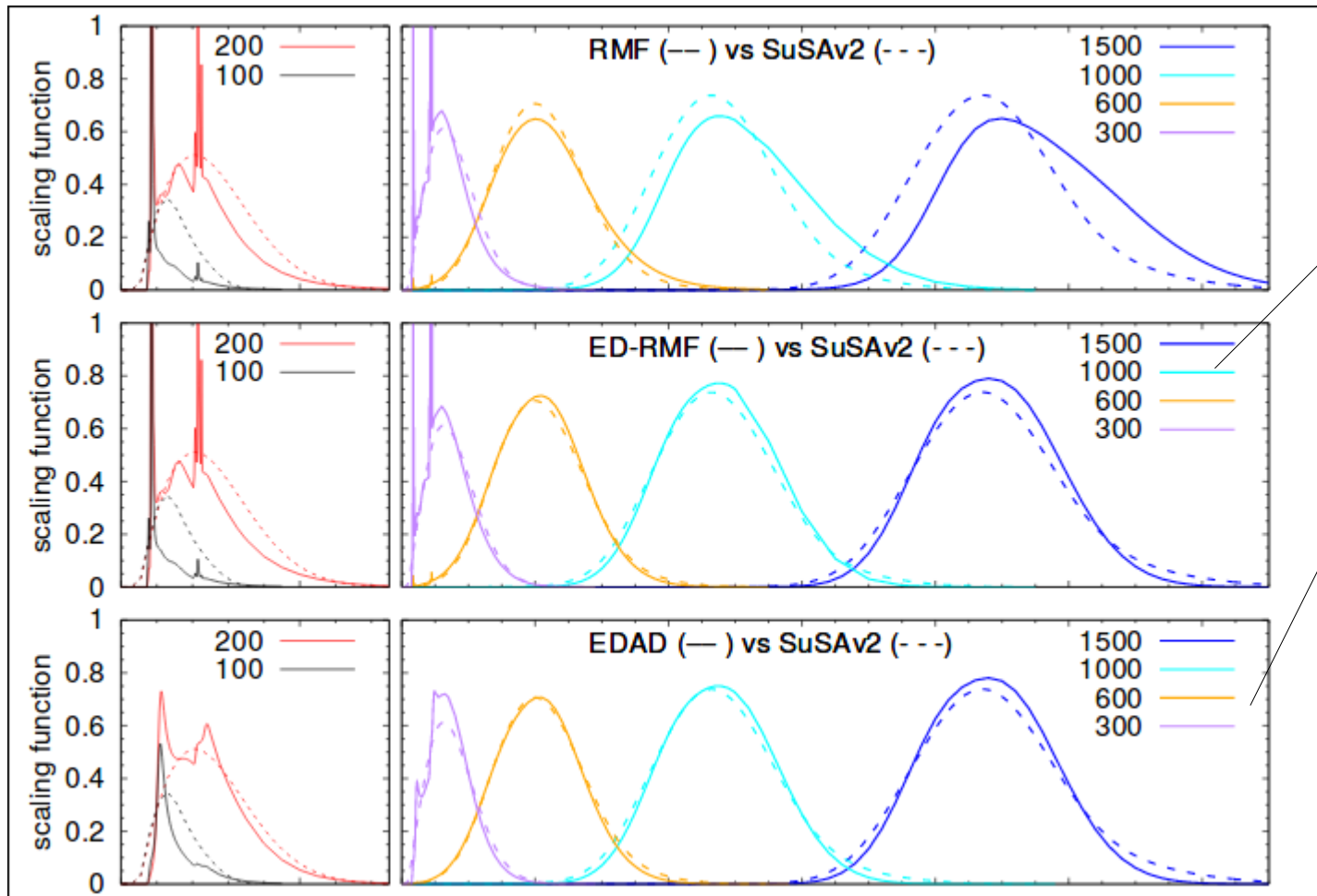


Dependence on  $q$   
and  $k_N$   
Becomes less  
important for high  
momenta

# Kinematic dependence

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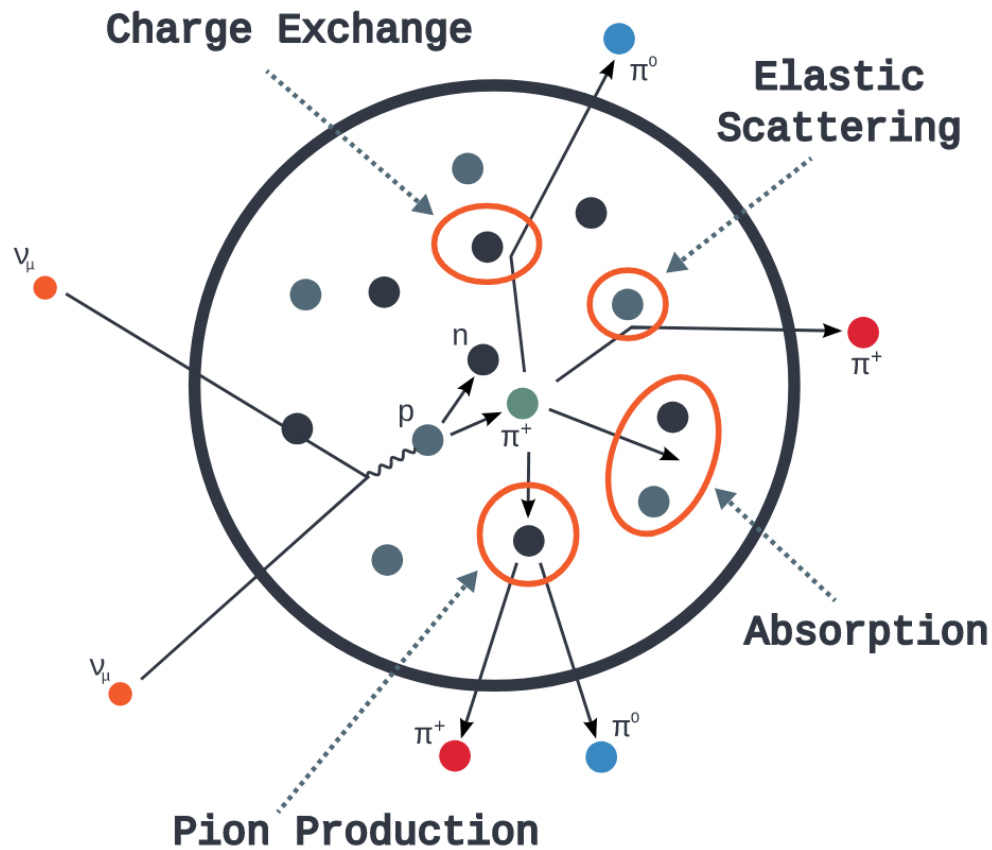
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Energy dependent potentials

Dependence on  $q$  and  $k_N$   
Becomes less important for high momenta

# Final state interactions



Distinction between:

## I. HARD FSI

Secondary interactions

(e.g. Absorption, charge exchange, ...)

Treated in Cascade model

## II. SOFT FSI

Influence of nuclear medium on  
energy-momentum of particle

Not included in Cascade

# Final state interactions

## I. HARD FSI

Secondary interactions

(e.g. Absorption, charge exchange, ...)

Treated in Cascade model

In principle: coupled channels

In practice : Optical potentials

## II. SOFT FSI

Influence of nuclear medium on  
energy-momentum of particle

Not included in Cascade

Imaginary part removes  
inelasticities from the final state

Inclusive  $\leftrightarrow$  Exclusive

Don't look at the final state  
All inelastic channels contribute

Look at one channel  
Flux is lost in inelasticities

# Final state interactions

## Inclusive $\leftrightarrow$ Exclusive

Don't look at the final state  
All inelastic channels contribute

Look at one channel  
Flux is lost in inelasticities

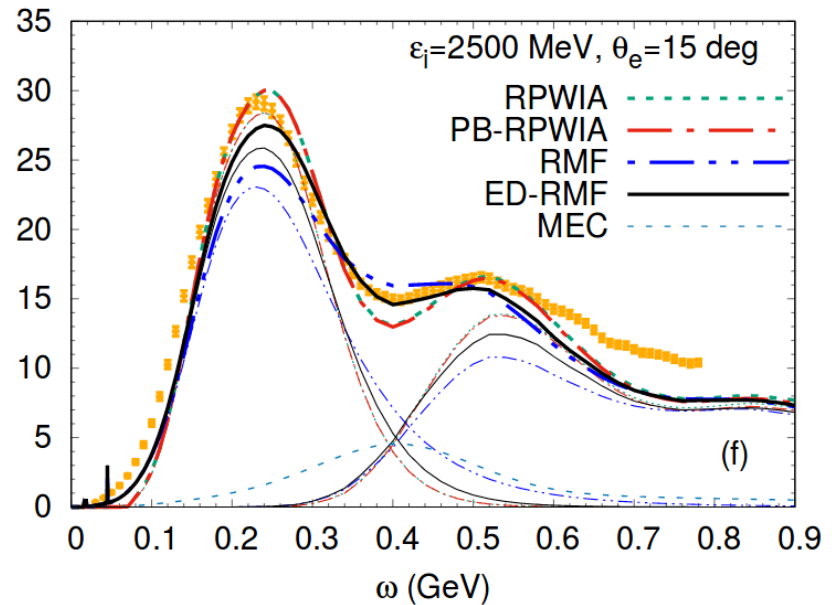
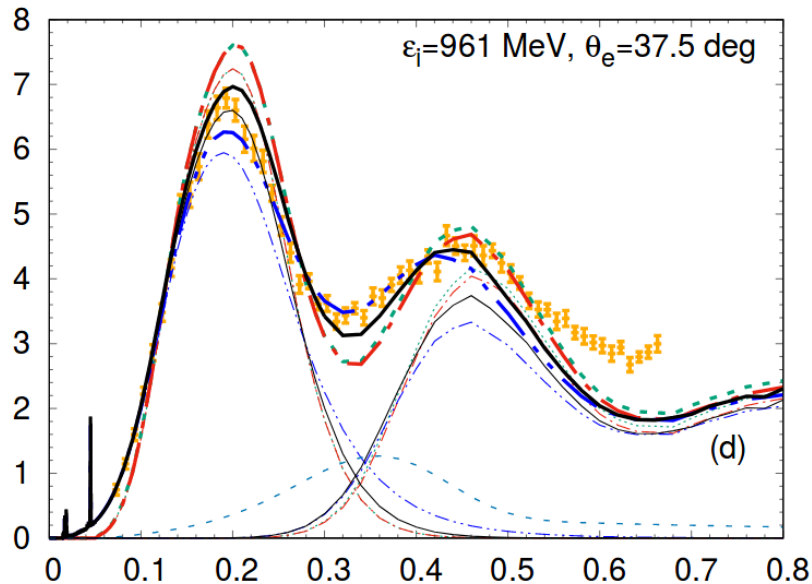
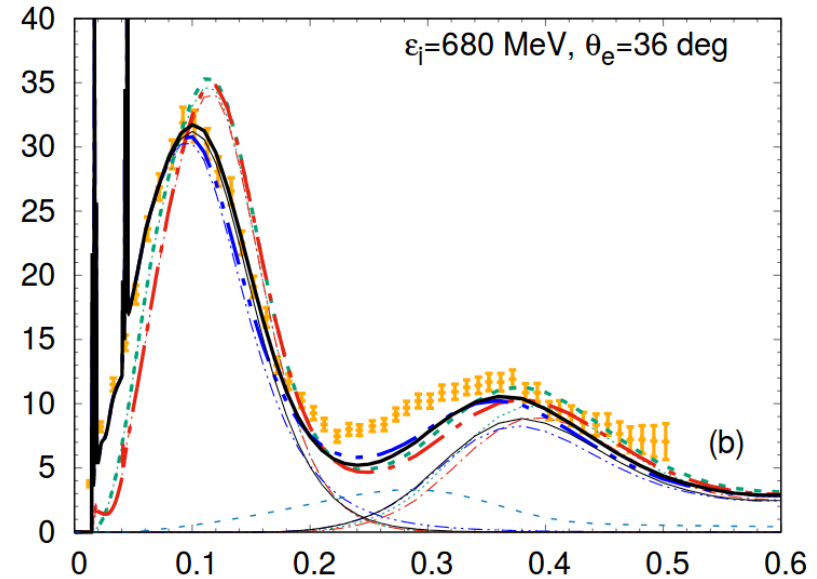
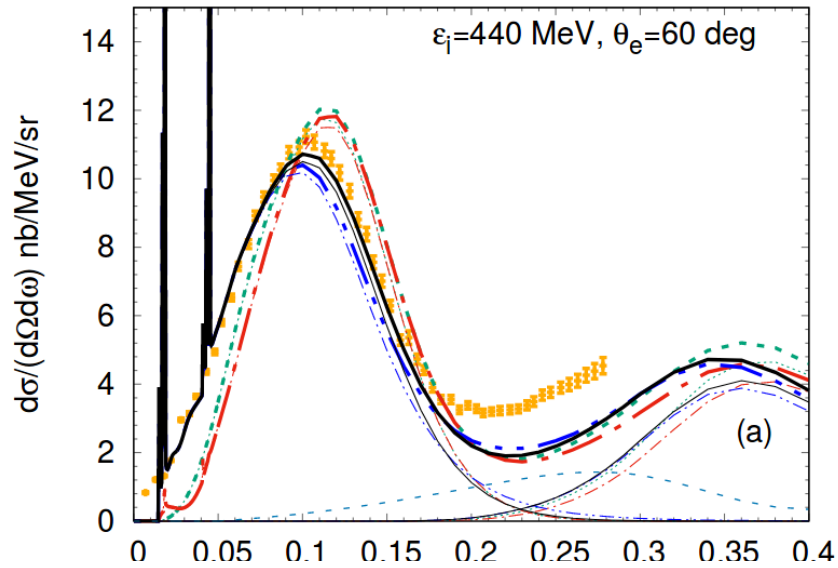
Potentials are energy dependent because  
Inelasticity grows as more channels open

RGF (A. Meucci, C. Giusti, et al. ) : recover flux lost in inelastic channels

RROP: Use real part of optical potential to conserve flux

ED-RMF: Phenomenological reduction of real RMF potential

# Distortion of the outgoing nucleon



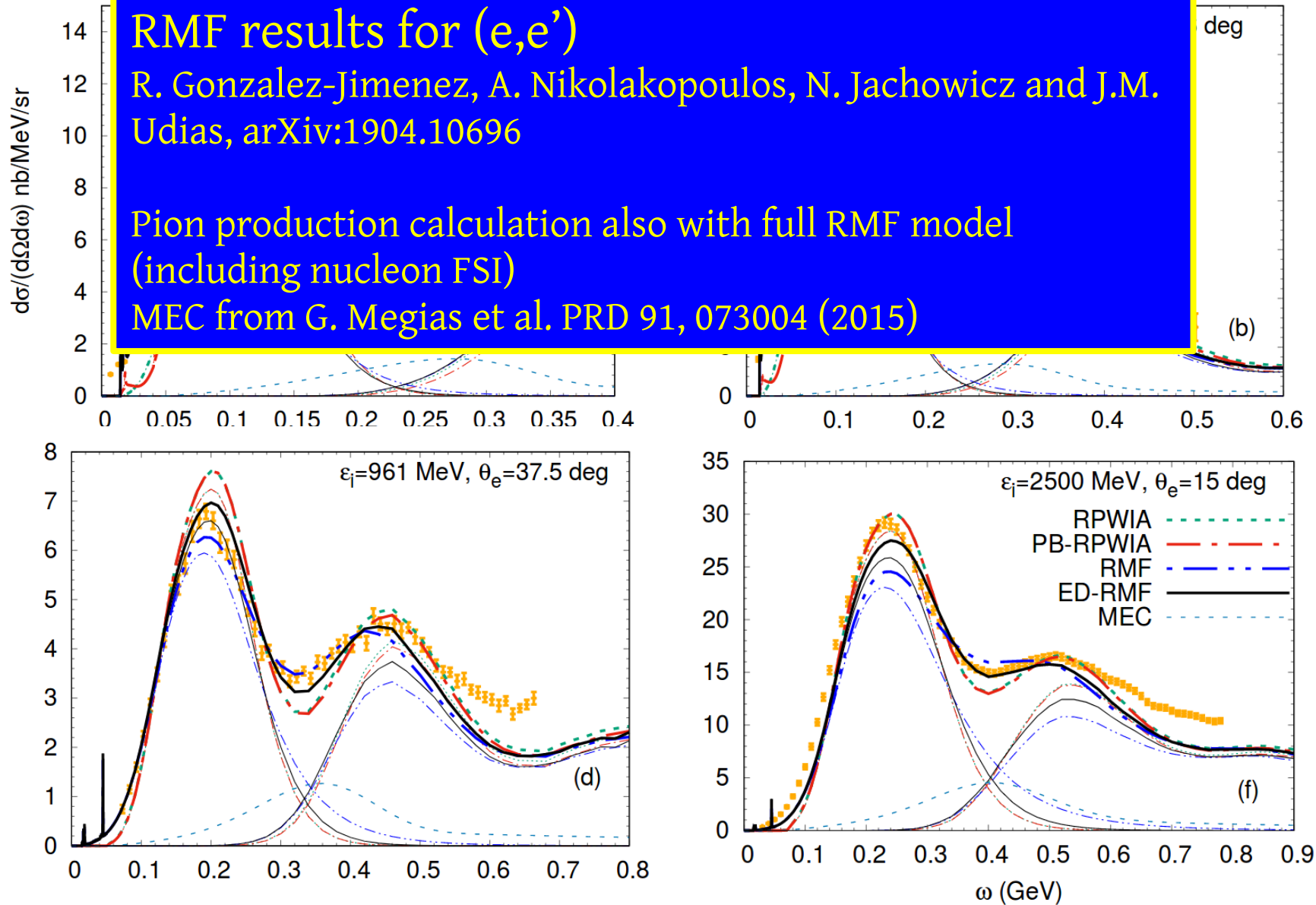
# Distortion of the outgoing nucleon

## RMF results for (e,e')

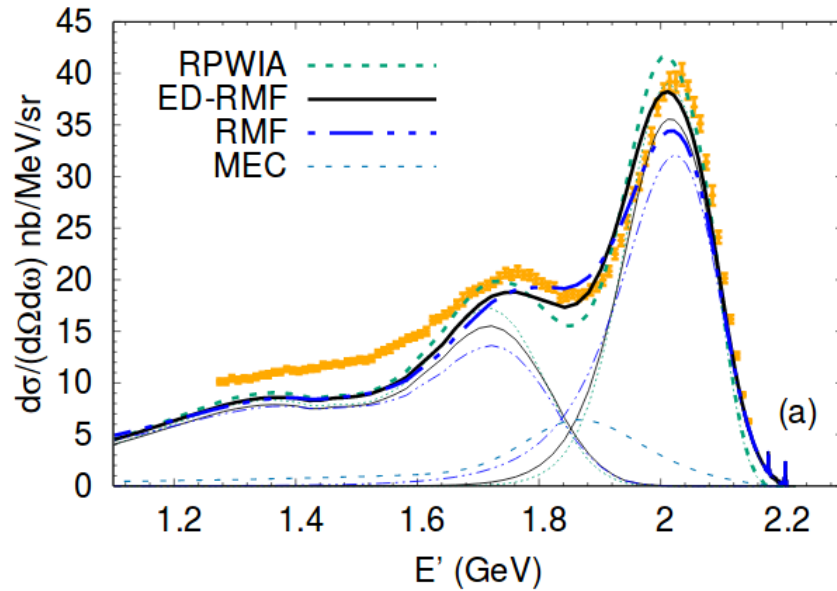
R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz and J.M. Udias, arXiv:1904.10696

Pion production calculation also with full RMF model (including nucleon FSI)

MEC from G. Megias et al. PRD 91, 073004 (2015)



# Distortion of the outgoing nucleon

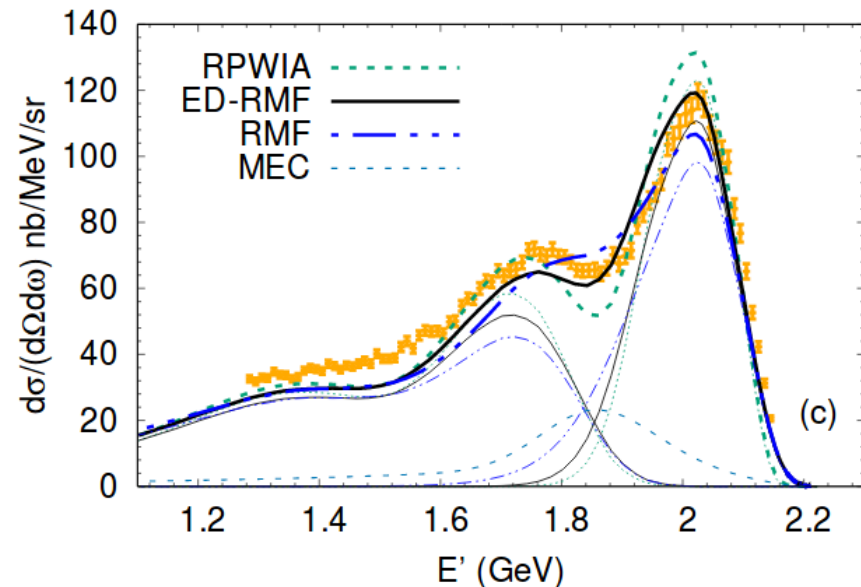
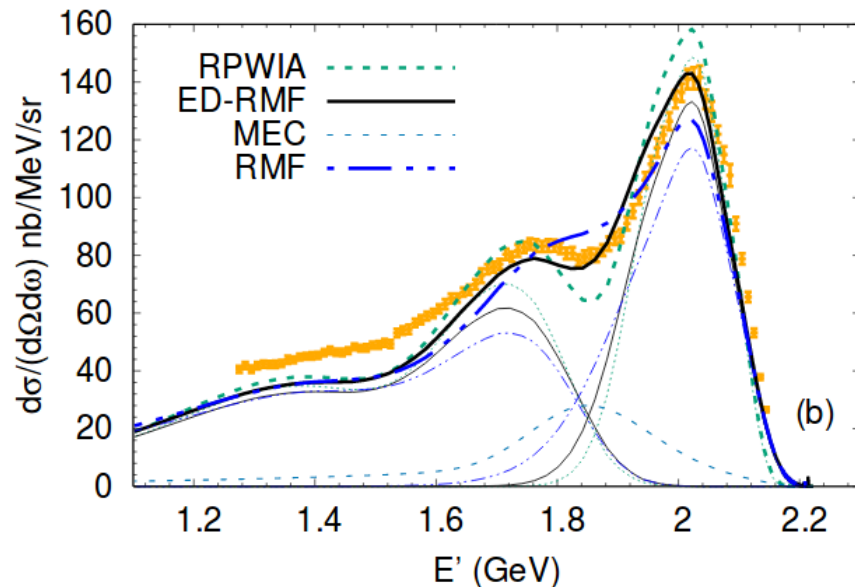


Carbon

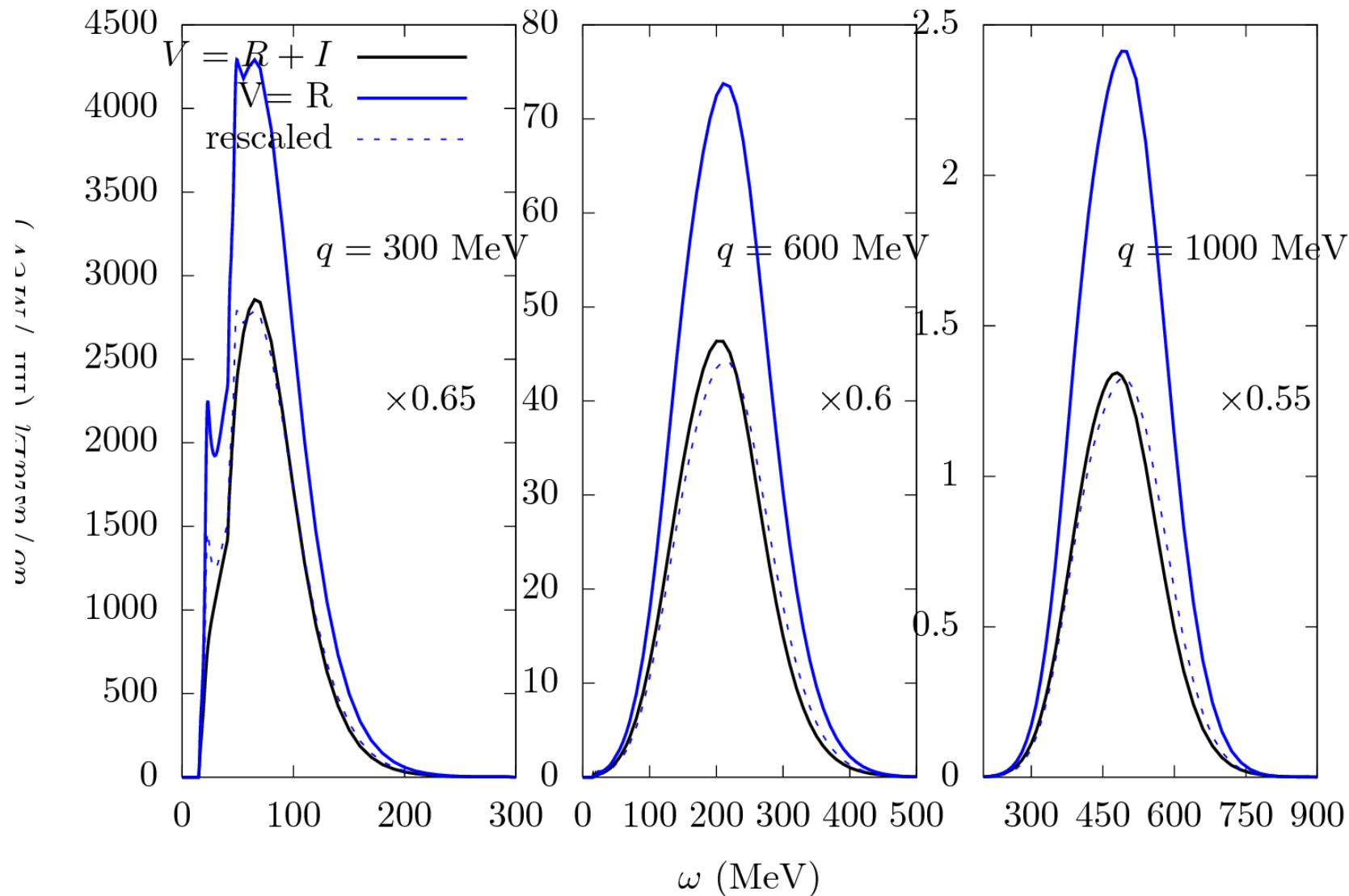
Arxiv:1909.07497

Titanium

Argon



# *$(e,e' p)$ and Final-State Interactions*

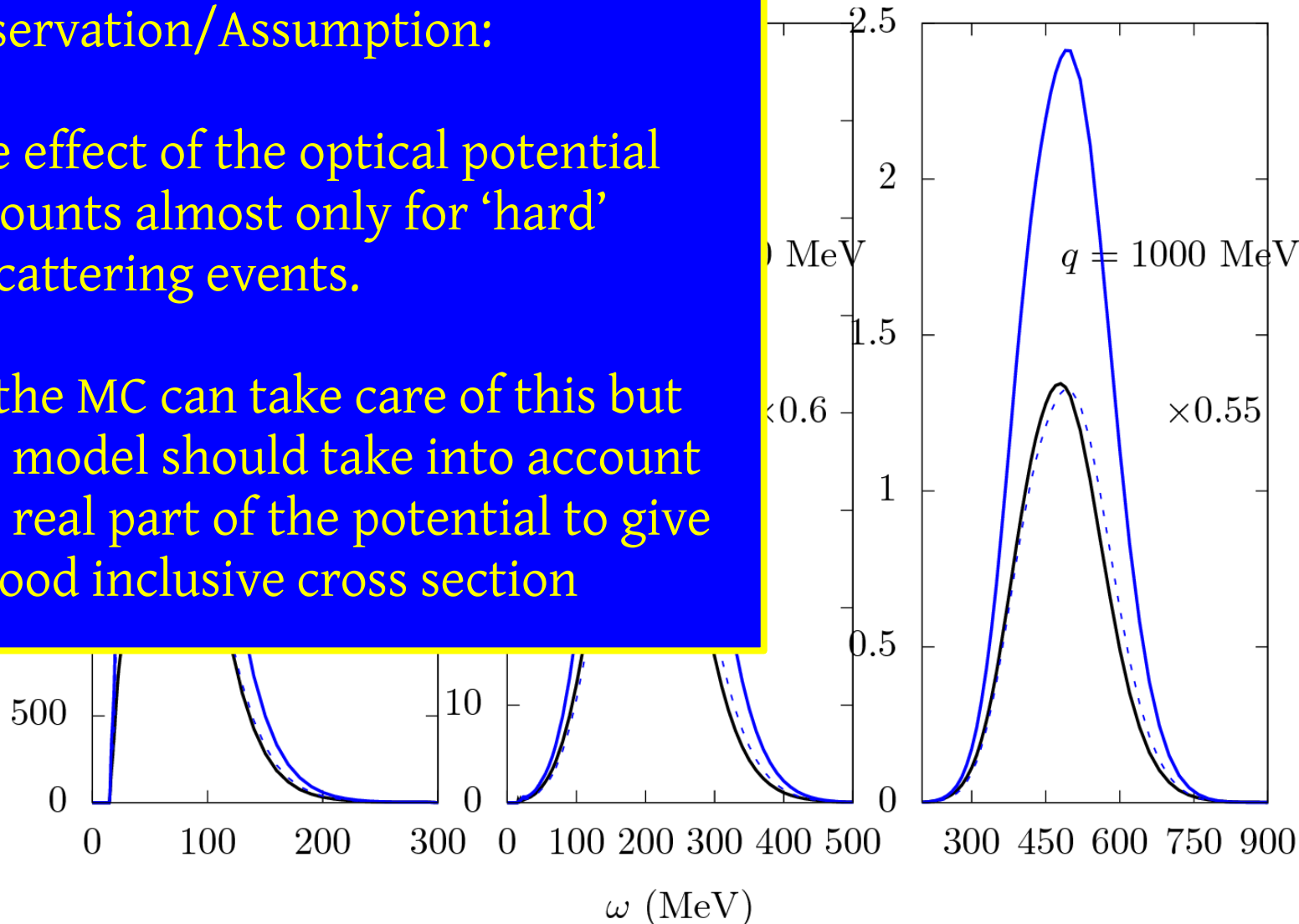


# $(e,e' p)$ and Final-State Interactions

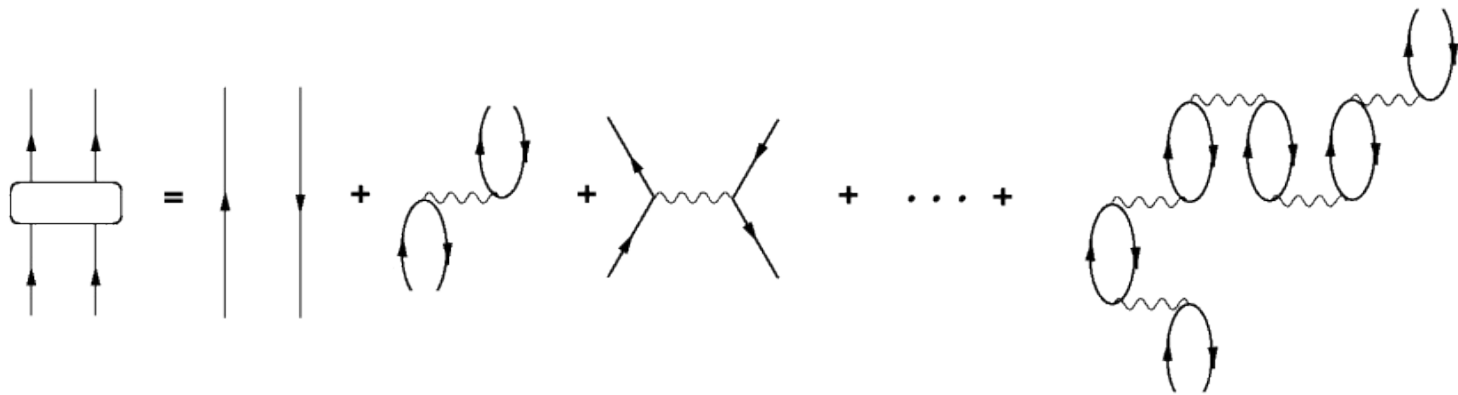
## Observation/Assumption:

The effect of the optical potential accounts almost only for 'hard' rescattering events.

So the MC can take care of this but the model should take into account the real part of the potential to give A good inclusive cross section

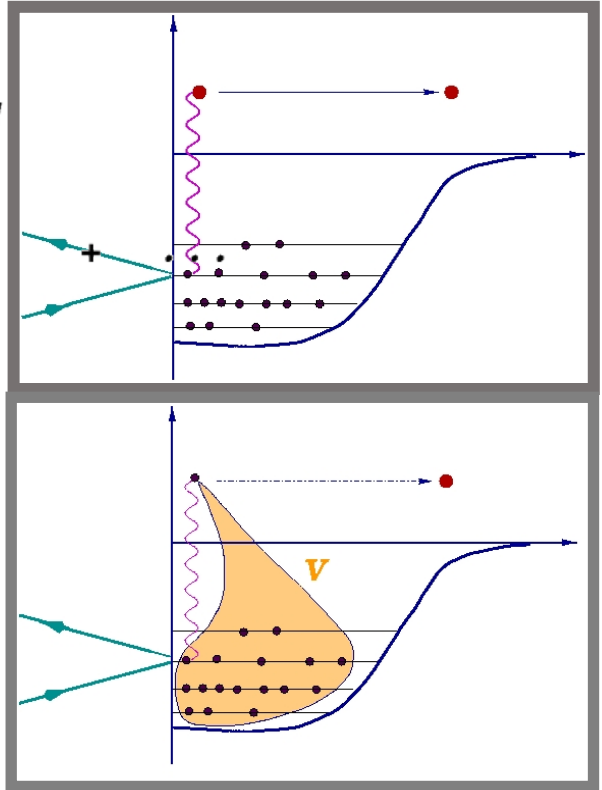


# Random Phase Approximation



Take into account long range nuclear excitations

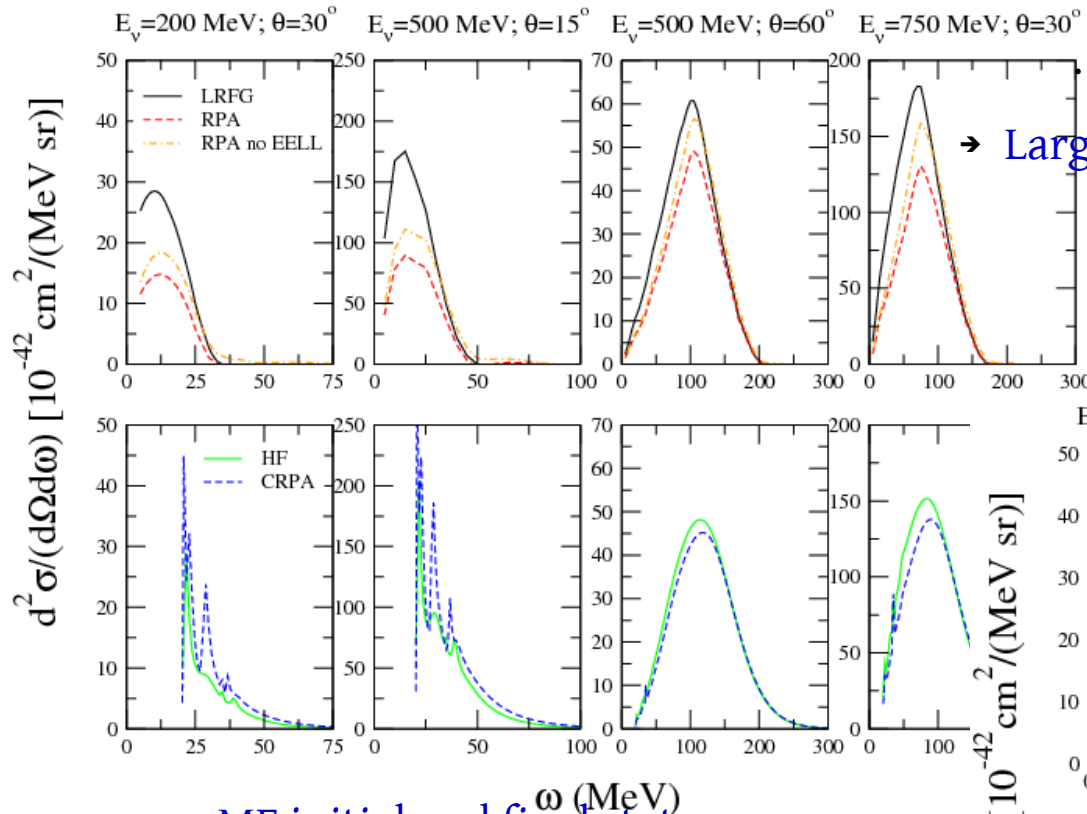
Every possible combination of excited SP states in nuclear medium



$$\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar} \int dx \int dx' \Pi^{(0)}(x_1, x; \omega) \tilde{V}(x, x') \Pi^{(RPA)}(x', x_2; \omega)$$

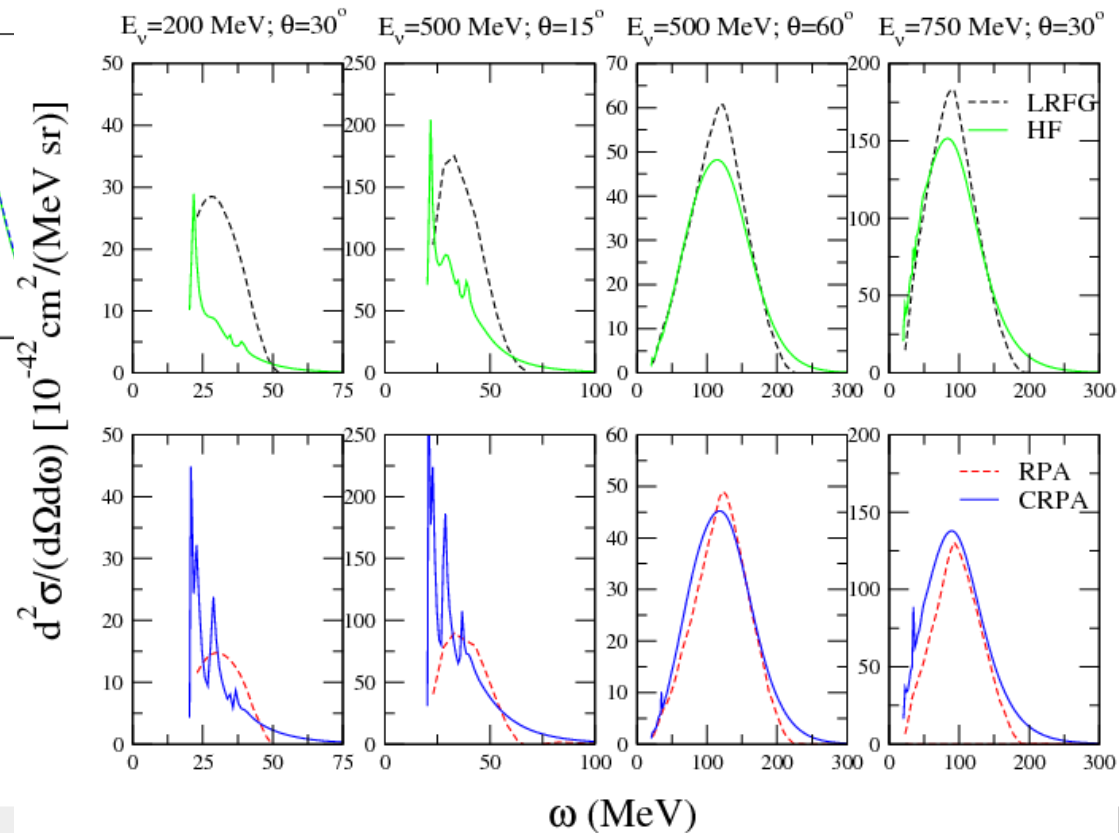
Mean field propagator

# Random Phase Approximation



- MF initial and final states
- Effect of RPA is smaller

Start from (basically) free initial and final states  
→ Large effect of RPA is needed to introduce interactions

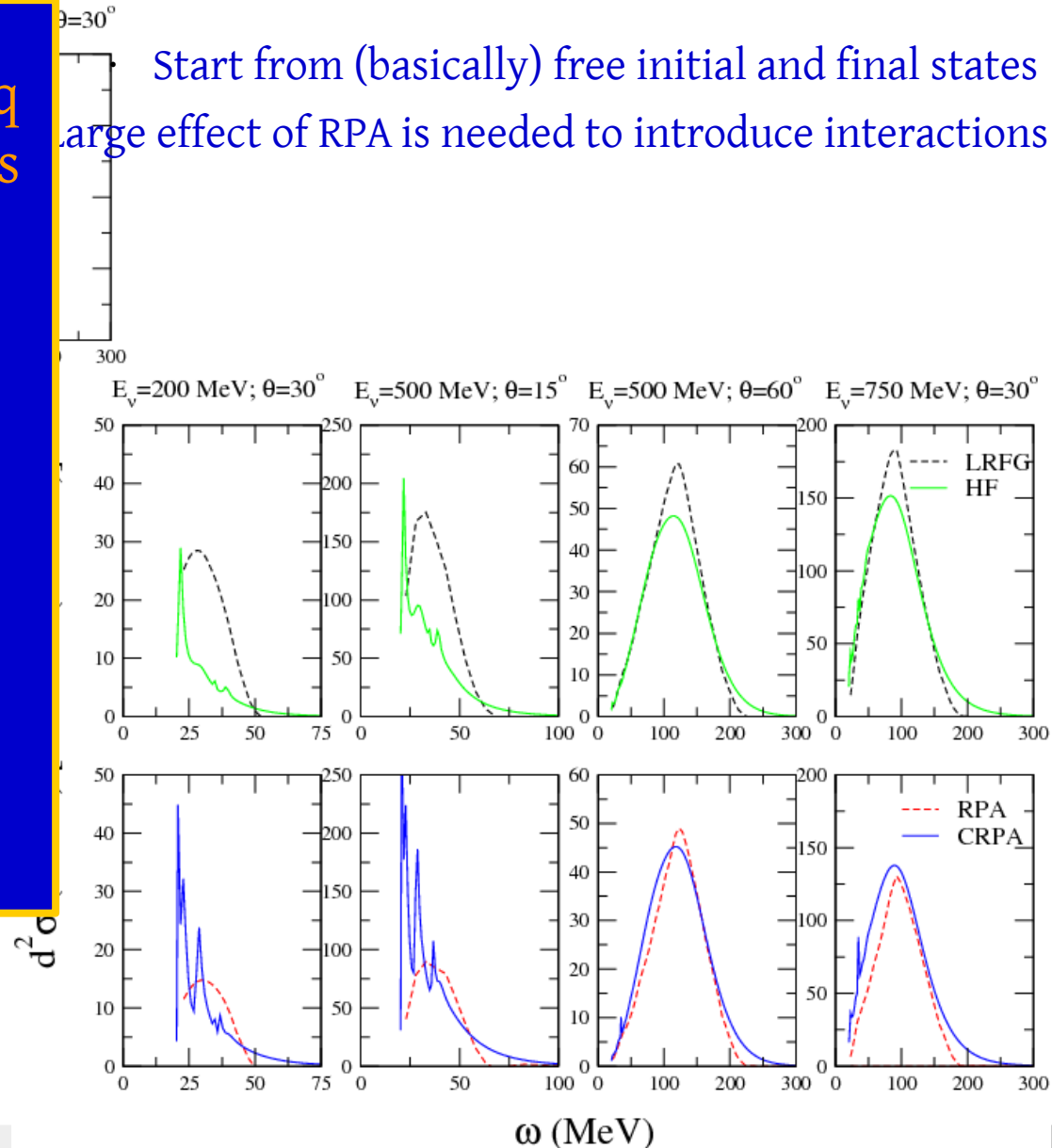


# Random Phase Approximation

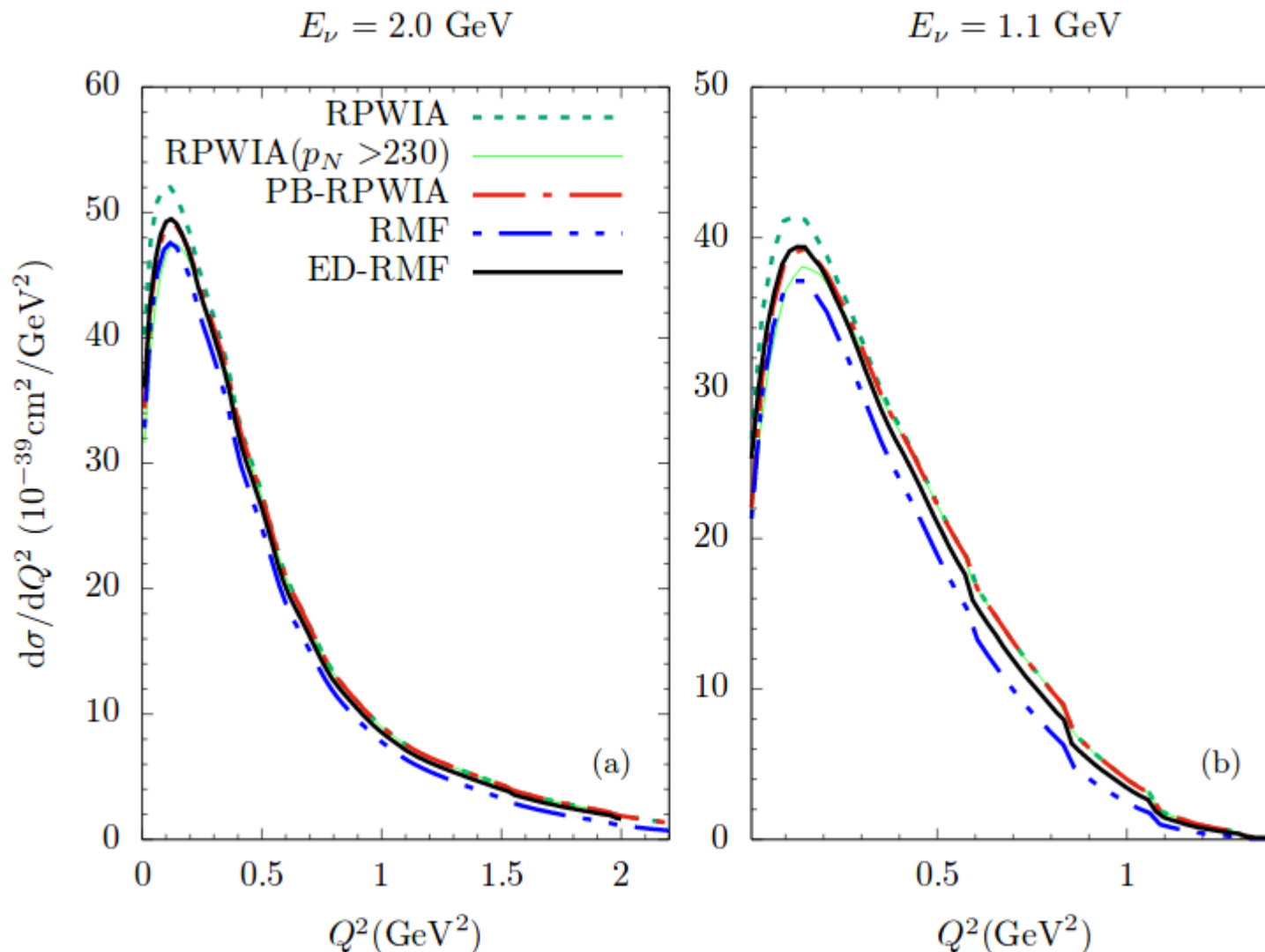
Largest reduction for low  $w$  and  $q$   
 $\rightarrow$  in QE scattering this corresponds  
 to low Nucleon momenta  
 $\rightarrow$  This is the region where  
 FSI is most important

Orthogonality  
 +  
 Spreading of wavefunction

Start from (basically) free initial and final states  
 large effect of RPA is needed to introduce interactions



# Nucleon FSI and $Q^2$ distributions

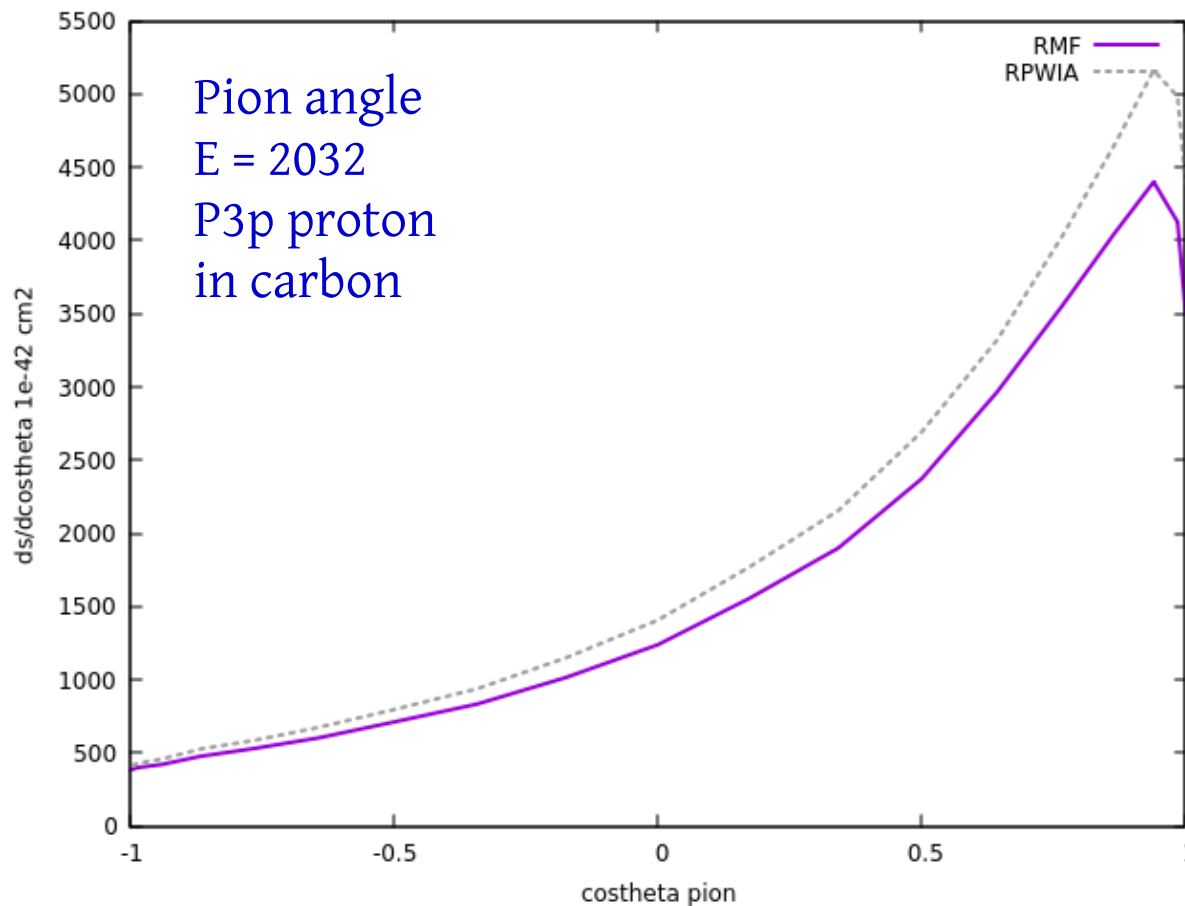


Reduction at low  $Q^2$   
Compared to RPWIA

Pion potential is still  
Missing, one expects  
A reduction in the  
same kinematic  
region

# Nucleon FSI and $Q^2$ distributions

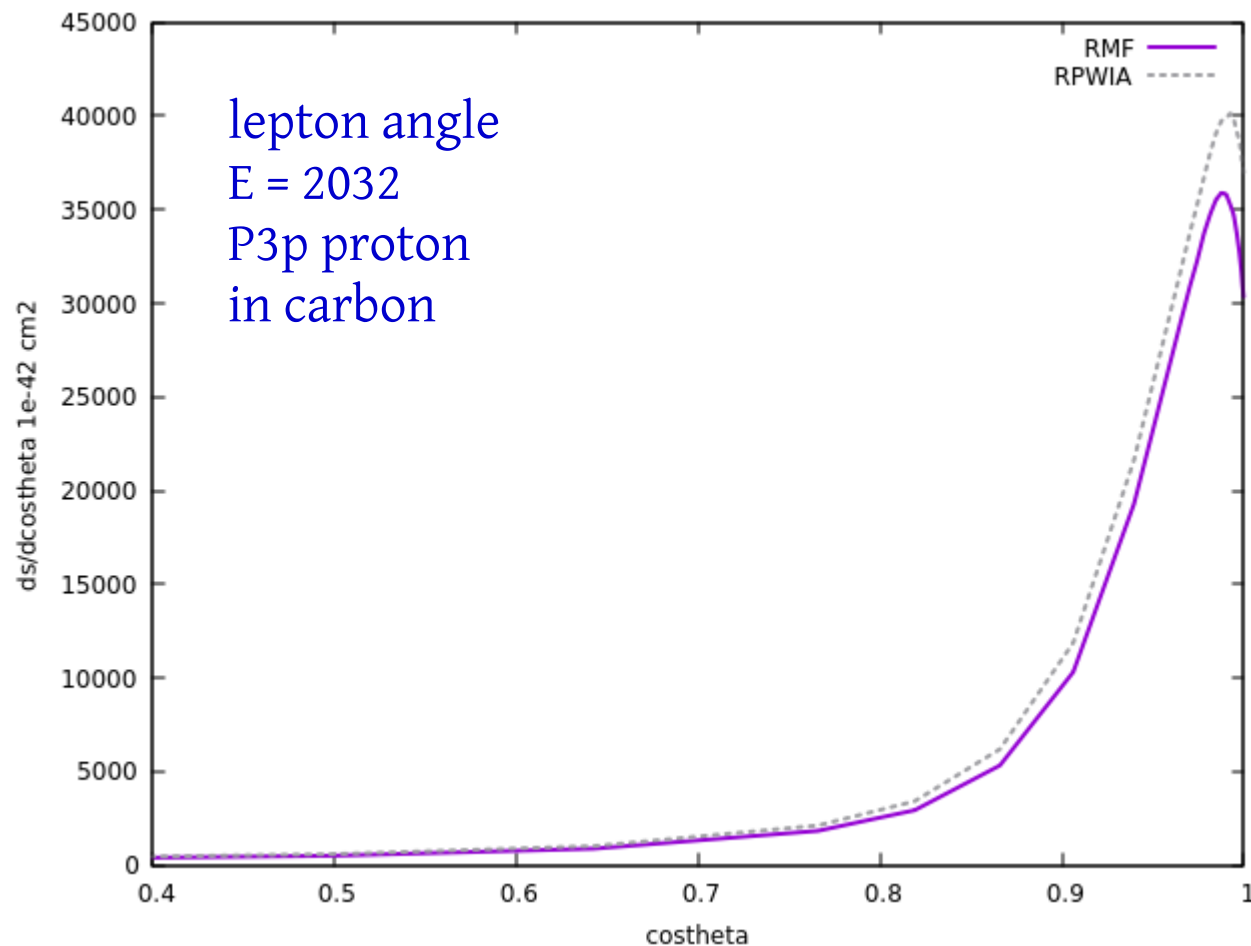
Does a deficit also show up in other distributions ?



Nucleon FSI leads to an overall reduction in pion angle  
Slightly stronger forward reduction

# Nucleon FSI and $Q^2$ distributions

Does a deficit also show up in other distributions ?



In lepton angle mostly  
Forward lepton  
reduction

# Conclusions

## I. Nucleon complexity

→ Angular distributions require higher dimensional sampling

## II. Nuclear complexity

→ Nuclear degrees of freedom require higher dimensional sampling

## III. Final state interaction

→ Consistently describing inclusive and exclusive signals is complicated

→ Nuclear effects depend on kinematics of outgoing hadrons

→ higher dimensional problems